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Study of some topological invariants of subdivided m^k graphs

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An m^k-graph of a graph G can be defined by taking m≥2 copies $G_1,\ldots, G_m$ of a graph G in which every vertex u_t of copy G_t is adjacent to a corresponding vertex v_s of copy G_s. An m^k-graph is represented by $m^k(G)$. In this research study, we discussed some degree based topological indices (connectivity indices) of subdivided m^k-graph generated by path graph and comb graph. The closed formulas for computing various degree based topological indices of subdivided m^k-graphs were presented.

KEYWORDS

Molecular graph; m^k-graph; path graph; comb graph; second zagreb index; augmented zagreb index.

Introduction

Topological indices are the numerical parameters in a graph. The first index is a Wiener index introduced by Wiener, describing the boiling point of the paraffin and defined as sum of the distance between all the pairs of vertices in G. The first and second Zagreb indices of a graph, denoted by $M_1(G)$ and $M_2(G)$ are the oldest, most popular and most extensively studied vertex-degree based topological indices. These indices were introduced by Gutman and Trinajstić in 1972, studying the structure dependency of the total electron energy. The augmented Zagreb index of graph G was proposed by Furtula *et al.* in 2010 [6]. This graph invariant has proved to be a valuable predictive index in the study of the heat of formation in octanes and heptanes (alkane). The atom bond connectivity index (ABC) a novel graph theoretical invariant based on the connectivity between atoms and bonds in a molecule, proposed by Ernesto Estrada et al. in 1998 [3]. ABC index has been used to describe the heats of formation of alkanes resulting in a good quantitative structure property and relationship (QSPR) model.

In this work, we work for the structure of some subdivide m^k graph. m^k graph is represent as $m^k(G)$. The family of m^k-graph was defined in 2017 by Ayache and Alameri [2]. An m^k graph of a graph G can be defined by taking m≥2 copies $G_1,\ldots, G_m$ of a graph G in which each vertex u_s of a graph G_s is adjacent to a correspondent vertex v_k of copy G_k.

We compute some topological indices such as Wiener index, F index, Hyper Wiener index, first and second Zagreb indices, ABC indices, augmented indices, Narumi-Katayama and GA indices etc. We compute all the following parameters for subdivided m^k-graph of Path and comb graph.

Degree based topological indices

Zagreb indices were proposed by I. Gutman and N. Trinajstić in 1972. Zagreb indices are one of the oldest kinds of degree based topological indices. There are generally two types of Zagreb indices which are usually used the first and second Zagreb indices. These indices are of great significance in chemical and mathematical terms. Zagreb indices are utilized in many research fields, and are commonly used for computation of topological indices.

First Zagreb [9] and second Zagreb indices [10]:

$$M_1(G)=\sum_{sk\in E(G)}(d_s+d_k)\ \&\ M_2(G)=\sum_{sk\in E(G)}(d_s\times d_k).$$

Reduced second Zagreb index is given by Gutman, Furtula and Ediz [7]. Its mathematical form is:

$$RM_2(G)=\sum_{sk\in E(G)}(d_s-1)(d_k-1).$$

Shirdel and Sayadi proposed Hyper Zagreb index in 2013 [14]. Defined as:

$$HM(G)=\sum_{sk\in E(G)}(d_s+d_k)^2.$$

Gutman and Furtula determined another index in 2015, known as the F-index which is given a name of forgotten topological index according to their proposition [5]. So F-index is defined as:

$$F(G)=\sum_{s\in V(G)}d^3(m)=\sum_{sk\in E(G)}(d^2(s)+d^2(k)).$$

Also, we can find it simply by putting values of HM(G) and M_2(G), respectively.

$$F(G)=HM(G)-2M_2(G).$$

Augmented Zagreb index represented as AZI(G) was introduced by Furtula and Graovac in 2010 [6]. It is defined as

$$AZI(G)=\sum_{sk\in E(G)}\left(\frac{d_sd_k}{d_s+d_k-2}\right)^3$$

Ghorbani, Azimi in (2012) introduced first and second multiple Zagreb indices [8], which are given as:

$$PM_1(G)=\prod_{k\in V(G)}(d_k)^2\ \&\ PM_2(G)=\prod_{sk\in E(G)}d_sd_k$$

Narumi and Katayama In (1984) for the first time proposed an index known as Narumi-Katayam index [11]. It is given as

$$NK(G)=\sqrt{PM_1G}$$

Fath-Tabar (in 2009) proposed the first and second Zagreb polynomials [1, 4, 11, 17-21], which are given as

$$ZG_1(G,x)=\sum_{sk\in E(G)}x^{d_s+d_k}\ \&\ ZG_2(G,x)=\sum_{sk\in E(G)}x^{d_sd_k}$$

where x is a dummy variable.

Atom-bond connectivity (ABC) index was proposed by Estrada and Torres in 1998 [3]. Atom-bond connectivity (ABC) index of a connected graph given as:

$$ABC(G)=\sum_{pq\in E(G)}\sqrt{\frac{d_p+d_q-2}{d_p\times d_q}}.$$

Vukičević and Furtula (2009) introduced the geometric-arithmetic index (GA) [15]. The geometric-arithmetic index (GA) defined as:

$$GA(G)=\sum_{pq\in E(G)}\frac{2\sqrt{d_p\times d_q}}{d_p+d_q}.$$

Connectivity indices of subdivided m^k-graph of path graph

Initially, we draw a simple subdivided graph. Then we construct subdivided m^k-graph of path graph as shown below. After that we generalize it for m and n, where m denotes number of copies and n denotes number of vertices.

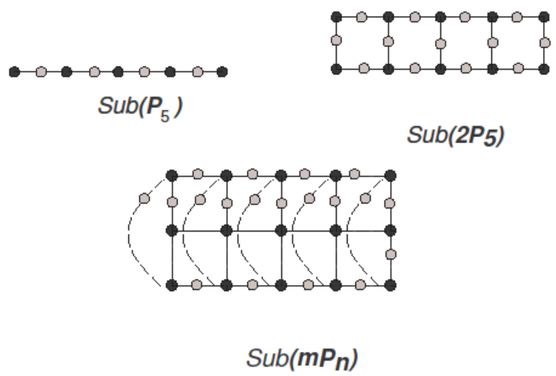

Figure 1 Some subdivided m^k-graphs of Path Graph for k=1

The dotted line in Figure 1 basically shows that this procedure continues up to m copies and n vertices in each copy, where light dark vertex between to vertex show the subdivision of edge in a graph.

TABLE 1 The edge partition of subdivided m^k-graph generated by P_n for k=1

(d_s,d_k)	(2,m)	(2, m+1)
Noof Edges	$2m^2$	[m(n-2)+(n-2)]m

TABLE 2 The vertex partition of subdivided m^k-graph generated by P_n for k=1

d_k	2	m	m+1
No of Vertices	$\frac{m(n-1)+m(m-1)n}{2}$	2m	m(n-2)

Theorem 2.1.1. Let G be a subdivided m^k-graph of path graph for k=1. Then first and second Zagreb indices given as

$$M_1(G)=m^3n+4m^2n-4m^2+3mn-6m,$$

$$M_2(G)=2m^3n+4m^2n-8m^2+2mn-4m.$$

Proof: FZ index is:

$$M_1(G)=\sum_{sk\in E(G)}(d_s+d_k).$$

By using Table 2. 1 in the equation we get

$$=(2m^2)(2+m)+[(n-2)m+(n-2)]m(2+m+1).$$

After simplifying it we have

$$M_1(G)=m^3n+4m^2n-4m^2+3mn-6m.$$

Now we calculate SZ index which is

$$M_2(G)=\sum_{sk\in E(G)}(d_s\times d_k).$$

Using Table 2. 1 in the above equation we

$$=(2m^2)[(2)(m)]+[(n-2)m+(n-2)]m[(2)(m+1)].$$

After simplifying it we get

$$M_2(G)=2m^3n+4m^2n-8m^2+2mn-4m.\blacksquare$$

Theorem 2.1.2. Let G be a subdivided m^k-graph of path graph for k=1. Then reduced second Zagreb index is computed as:

$$RM_2(G)=m^3n+m^2n-4m^2.$$

Proof: RSZ index is:

$$RM_2(G)=\sum_{sk\in E(G)}(d_s-1)(d_k-1).$$

By using Table 2. 1 in the given equation

$$=2m^2[(2-1)(m-1)]+[(n-2)m+(n-2)]m[(2-1)(m+1-1)].$$

After simplifying it we get

$$RM_2(G)=m^3n+m^2n-4m^2.\blacksquare$$

Theorem 2.1.3. Let G be a subdivided m^k-graph of path graph for k=1. Then Hyper Zagreb index is:

$$HM(G)=m^4n+7m^3n-6m^3+15m^2n-22m^2+9mn-18m.$$

Proof: HZ index is given

$$HM(G)=\sum_{sk\in E(G)}(d_s+d_k)^2$$

By using Table 2. 1 we get

$$=2m^2[2+m]^2+[(n-2)m+(n-2)]m[2+m+1]^2$$

After simplifying it we have our result

$$HM(G)=m^4n+7m^3n-6m^3+15m^2n-22m^2+9mn-18m.\blacksquare$$

Theorem 2.1.4. Let G be a subdivided m^k-graph of path graph for k=1. Then F-index can b calculated as:

$$F(G)=m^4n+3m^3n-6m^3+7m^2n-6m^2+5mn-10m.$$

Proof: F-index is given *F(G)=M₃(G)-2M₂(G)*. Here $M_3(G)$ is HM(G). Now using theorem 2.1.1. and theorem 2.1.3. We get

$$=[m^4n+7m^3n-6m^3+15m^2n-22m^2+9mn-18m]$$

$$-2[2m^3n+4m^2n-8m^2+2mn-4m].$$

After simplifying it we have our result

$$F(G)=m^4n+3m^3n-6m^3+7m^2n-6m^2+5mn-10m.\blacksquare$$

Theorem 2.1.5. Let G be a subdivided m^k-graph of path graph for k=1. Then augmented Zagreb index can be Calculated as: $AZI(G)=8m^2n+8mn-16m$.

Proof: AZ index is

$$AZI(G)=\sum_{sk\in E(G)}\left(\frac{d_sd_k}{d_s+d_k-2}\right)^3.$$

By using Table 2. 1 we get

$$=(2m^2)\left[\frac{(2)(m)}{2+m-2}\right]^3+[(n-2)m+(n-2)]m\left[\frac{(2)(m+1)}{2+m+1-2}\right]^3$$

After simplifying it we have our result

$$AZI(G)=8m^2n+8mn-16m.\blacksquare$$

Theorem 2.1.6. Let G be a subdivided m^k-graph of path graph for k=1. Then first and second multiple Zagreb indices (MZI) are:

$$PM_1(G)=4m^8n^2-8m^8n+12m^7n^2-32m^7n$$
$$+12m^6n^2+16m^7-40m^6n+4m^5n^2+32m^6-16m^5$$

Proof: FMZ index is given as

$$PM_1(G)=\prod_{k\in V(G)}(d_k)^2.$$

By using Table 2.2 in the above equation we get

$$=(2^2)\times\left(m(n-1)+\frac{m(m-1)}{2}n\right)$$
$$\times(m)^2\times(2m)\times(m+1)^2\times(m(n-2))$$

After simplifying we get our result:

$$PM_1(G)=4m^8n^2-8m^8n+12m^7n^2-32m^7n$$
$$+12m^6n^2+16m^7-40m^6n+4m^5n^2+32m^6-16m^5$$

Now SMZ index is

$$PM_2(G)=\prod_{sk\in E(G)}d_sd_k$$

By using Table 2. 1 in above equation

$$=(2)(m)\times(2m^2)\times 2(m+1)\times[(n-2)m+(n-2)]m.$$

After simplifying it, we have our result:

$$PM_2(G)=8m^6n-16m^6+16m^5n-32m^5+8m^4n-16m^4.\blacksquare$$

Theorem 2.1.7. Let G be a subdivided m^k-graph of path graph for k=1. Then Narumi-Katayama (NK) index can b calculated as

$$NK(G)=\sqrt{\begin{matrix}4m^8n^2-8m^8n+12m^7n^2-32m^7n+12m^6n^2\\+16m^7-40m^6n+4m^5n^2+32m^6-16m^5\end{matrix}}$$

Proof: NK index is defines as

$$NK(G)=\sqrt{PM_1(G)}$$

By using theorem 2.1.6 in this relation

$$NK(G)=\sqrt{\alpha}$$

where

$$\alpha=4m^8n^2-8m^8n+12m^7n^2-32m^7n+12m^6n^2$$
$$+16m^7-40m^6n+4m^5n^2+32m^6-16m^5$$

After simplifying we get our result

$$NK(G)=2\sqrt{\beta}$$

where

$$\beta=m^8n^2-2m^8n+3m^7n^2-8m^7n+3m^6n^2.\blacksquare$$
$$+4m^7-10m^6n+m^5n^2+8m^6-4m^5$$

Theorem 2.1.8. Let G be a subdivided m^k-graph of path graph for k=1. Then first Zagreb polynomial is:

$$ZG_1(G,x)=(2m^2)\times x^{2+m}+[m^2n-2m^2+mn-2]\times x^{m+3}.$$

Proof: FZ polynomial is

$$ZG_1(G,x)=\sum_{sk\in E(G)}x^{d_s+d_k}.$$

By using Table 2. 1 we get

$$=(2m^2)\times x^{2+m}+[(n-2)m+(n-2)]m\times x^{2+m+1}$$

After simplifying we have our result

$$ZG_1(G,x)=(2m^2)x^{2+m}+(m^2n-2m^2+mn-2)x^{m+3}.\blacksquare$$

Theorem 2.1.9. Let G be a subdivided m^k-graph of path graph for k=1. Then second Zagreb polynomial is:

$$ZG_2(G,x)=(2m^2)x^{2m}+(m^2n-2m^2+mn-2m)x^{2m+2}.$$

Proof: SZ polynomial is given

$$ZG_2(G,x)=\sum_{sk\in E(G)}x^{d_sd_k}.$$

By using Table 2. 1 in the above equation we get

$$=(2m^2)\times x^{2m}+[(n-2)m+(n-2)]m\times x^{2(m+1)}.$$

After simplifying we get

$$ZG_2(G,x)=(2m^2)x^{2m}+(m^2n-2m^2+mn-2m)x^{2m+2}.\blacksquare$$

Theorem 2.1.10. Let G be an m^k-graph of path graph for k=1. Then ABC-index is computed as:

$$ABC(G)=\frac{1}{\sqrt{2}}(m^2n+mn-2m).$$

Proof: ABC index is given as

$$ABC(G)=\sum_{pq\in E(G)}\sqrt{\frac{d_p+d_q-2}{d_p\times d_q}}$$

By using Table 2. 1 we get

$$=(2m^2)\sqrt{\frac{2+m-2}{(2)(m)}}+[(n-2)m+(n-2)]m\sqrt{\frac{2+m+1-2}{2(m+)}}$$

After simplifying we get our result

$$ABC(G)=\frac{1}{\sqrt{2}}(m^2n+mn-2m).\blacksquare$$

Theorem 2.1.11. Let G be a subdivided m^k-graph of path graph for k=1. Then GA-index is computed as:

$$GA(G)=(2m^2)\frac{2\sqrt{2m}}{2+m}+(m^2n-2m^2+mn-2m)\frac{2\sqrt{2m+2}}{m+3}.$$

Proof: GA-index is $GA(G)=\sum_{pq\in E(G)}\frac{2\sqrt{d_p\times d_q}}{d_p+d_q}$

By using Table 2. 1 in the above equation

$$=(2m^2)\frac{2\sqrt{(2)(m)}}{2+m}+(m^2n-2m^2+mn-2m)\frac{\sqrt{(2)(m+1)}}{2+m+1}.$$

After simplifying we get our result

$$GA(G)=(2m^2)\frac{2\sqrt{2m}}{2+m}+(m^2n-2m^2+mn-2m)\frac{2\sqrt{2m+2}}{m+3}.\blacksquare$$

Connectivity indices of subdivided mk-graph of comb graph

First, we draw a simple comb graph as shown in figure then we will construct a subdivided mk-graph of comb graph as shown $2P_4 \odot K_1$. After this we generalize it for m and n where m denotes number of copies and n denotes number of vertices.

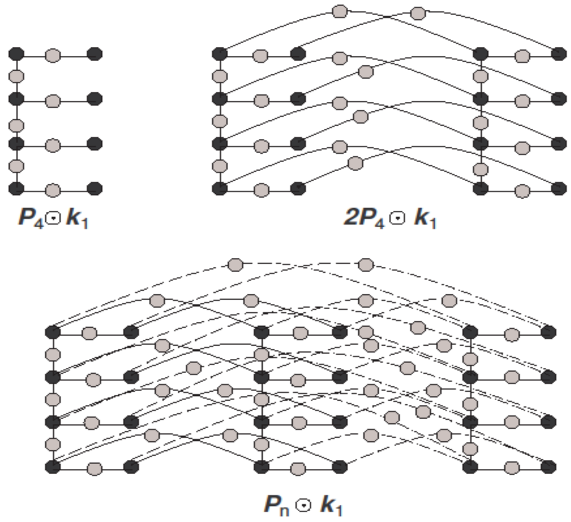

FIGURE 2 Some subdivided mk-graph of Comb Graph for k=1

The dotted line in last figure shows that this procedure is continues up to m copies and n vertices, where light dark vertexes between to vertex show the subdivision of edge in a graph.

TABLE 3 The edge partition of subdivided mk-graph generated by Comb graph for k=1

(d_s, d_k)	(2, m)	(2, m+1)	(2, m+2)
No of Edges	m^2n	$2m(m+1)$	$m(n-2)(m+2)$

TABLE 4 The vertex partition of subdivided mk-graph generated by Comb graph for k=1

d_k	2	m	m+1	m+2
No of Vertices	$m(2n-1)+\frac{m(m-1)2n}{2}$	mn	2m	m(n-2)

Theorem 2.2.1. Let G be a subdivided mk-graph of comb graph for k=1. Then first and second Zagreb indices are computed as:

$$M_1(G)=2m^3n+8m^2n-4m^2+8mn-10m,$$

$$M_2(G)=4m^3n+8m^2n-8m^2+8mn-12m.$$

Proof: FZ index is

$$M_1(G)=\sum_{sk\in E(G)}(d_s+d_k)$$

By using Table 2.3 we obtain

$$=(m^2n)(2+m)+2m(m+1)(2+m+1)$$
$$+m(n-2)(m+2)(2+m+2)$$

After simplifying it we get

$$M_1(G)=2m^3n+8m^2n-4m^2+8mn-10m.$$

Now SZ index is

$$M_2(G)=\sum_{sk\in E(G)}(d_s\times d_k)$$

We apply Table 2.3 in the above equation

$$=(m^2n)((2)(m))+2m(m+1)((2)(m+1))$$
$$+m(n-2)(m+2)((2)(m+2))$$

After simplifying we get our result

$$M_2(G)=4m^3n+8m^2n-8m^2+8mn-12m. \blacksquare$$

Theorem 2.2.2. Let G be a subdivided m^k-graph of comb graph for k=1. Then reduced second Zagreb index computed as: $RM_2(G)=2m^3n+2m^2n-4m^2+2mn-4m.$

Proof: RSZ index is

$$RM_2(G)=\sum_{sk\in E(G)}(d_s-1)(d_k-1)$$

By using Table 2.3 we get

$$=(m^2n)((2-1)(m-1))+(2m^2+2m)((2-1)(m+1-1))$$
$$+m(n-2)(m+2)((2-1)(m+2-1))$$

After simplifying we get

$$RM_2(G)=2m^3n+2m^2n-4m^2+2mn-4m. \blacksquare$$

Theorem 2.2.3. Let G be a subdivided m^k-graph of comb graph for k=1. Then Hyper Zagreb index computed as: $HM(G)=2m^4n+14m^3n-6m^3+36m^2n-34m^2+32mn-46m.$

Proof: HZ index is given as

$$HM(G)=\sum_{sk\in E(G)}(d_s+d_k)^2$$

By using Table 2.3 we get

$$=(m^2n)(m+2)^2+(2m^2+2m)(m+3)^2$$
$$+(m(n-2)(m+2))(m+4)^2$$

After simplifying we have our result:

$$HM(G)=2m^4n+14m^3n-6m^3 \blacksquare$$
$$+36m^2n-34m^2+32mn-46m.$$

Theorem 2.2.4. Let G be an mk-graph of comb graph for k=1. Then F-index is computed as:

$$F(G)=2m^4n+6m^3n-6m^3+20m^2n-18m^2+16mn-22m.$$

Proof: F-index is $F(G)=M_3(G)-2M_2(G)$.

By using theorem 2.2.1 and 2.2.3 we get

$$=(2m^4n+14m^3n-6m^3+36m^2n-34m^2+32mn-46m)$$
$$-2[4m^3n+8m^2n-8m^2+8mn-12m].$$

After simplifying we get

$$HM(G)=2m^4n+14m^3n-6m^3+36m^2n-34m^2+32mn-46m. \blacksquare$$

Theorem 2.2.5. Let G be a subdivided m^k-graph of comb graph for k=1. Then augmented Zagreb index is:

AZI(G)=16m²n+16mn-16m

Proof: AZ index is

$$AZI(G)=\sum_{sk\in E(G)}[\frac{d_sd_k}{d_s+d_k-2}]^3$$

By using Table 2.3 we get

$$=(m^2n)[\frac{2m}{2+m-2}]^3+(2m^2+2m)[\frac{2(m+1)}{2+m+1-2}]^3$$
$$+((m(n-2)(m+2)))[\frac{2(m+2)}{2+m+2-2}]^3$$

After simplifying we get

$$AZI(G)=16m^2n+16mn-16m. \blacksquare$$

Theorem 2.2.6. Let G be a subdivided m^k-graph of comb graph for k=1. Then first and second multiple Zagreb indices are:

$$PM_1(G)=8m^{12}n^2+32m^{11}n^2-16m^{11}n+8m^{10}n^2-64m^{10}n-112m^9n^2$$
$$-16m^9n-160m^8n^2+224m^8n-64m^7n^2+320m^7n+128m^6.$$
$$PM_2(G)=16m^9n^2-32m^9n+96m^8n^2-192m^8n+208m^7n^2$$
$$-416m^7n+192m^6n^2-384m^6n+64m^5n^2-128m^5n.$$

Proof: FMZ index is

$$PM_1(G)=\prod_{k\in V(G)}(d_k)^2$$

By using Table 2. 4 in the above equation we get

$$=[(m(n-2))+\frac{m(m-1)(2n)}{2}]\times(2)^2]\times[(mn)(m^2)]$$
$$\times[(2m)(m+1)^2]\times[m(m-2)(m+2)^2]$$

After simplifying we get our result

$$PM_1(G)=8m^{12}n^2+32m^{11}n^2-16m^{11}n+8m^{10}n^2-64m^{10}n-112m^9n^2$$
$$-16m^9n-160m^8n^2+224m^8n-64m^7n^2+320m^7n+128m^6.$$

Now SMZ index is:

$$PM_2(G)=\prod_{sk\in E(G)}d_sd_k.$$

By using Table 2.3 in above equation

$$=[(2)(m)\times(m^2n)]\times[(2)(m+1)\times(2m)(m+1)]$$
$$\times[(2)(m+2)\times(m(n-2)(m+2))]$$

After simplifying we have our result:

$$PM_2(G)=16m^9n^2-32m^9n+96m^8n^2-192m^8n+208m^7n^2 \blacksquare$$
$$-416m^7n+192m^6n^2-384m^6n+64m^5n^2-128m^5n.$$

Theorem 2.2.7. Let G be a subdivided m^k-graph of comb graph for k=1. Then Narumi-Katayama index is computed as: $NK(G)=\sqrt{\beta}$ where

$$\beta=8m^{12}n^2+32m^{11}n^2-16m^{11}n+8m^{10}n^2-64m^{10}n-112m^9n^2-16m^9n$$
$$-160m^8n^2+224m^8n-64m^7n^2+320m^7n+128m^6n$$

Proof: NK index is defined as $NK(G)=\sqrt{PM_1(G)}$. By using theorem 2.2.6. in above equation we get After simplifying we get our result:

$$\beta = 8m^{12}n^2 + 32m^{11}n^2 - 16m^{11}n + 8m^{10}n^2 - 64m^{10}n - 112m^9n^2 - 16m^9n - 160m^8n^2 + 224m^8n - 64m^7n^2 + 320m^7n + 128m^6n$$

where $NK(G) = \sqrt{\beta}$..■

Theorem 2.2.8. Let G be a subdivided m^k-graph of comb graph for k=1. Then first Zagreb polynomial is:

$$ZG_1(G,x) = (m^2n)x^{m+2} + (2m^2+2m))x^{m+3} + (m^2n + 2mn - 2m^2 - 4m)x^{m+4}.$$

Proof: FZ polynomial is defined as

$$ZG_1(G,x) = \sum_{sk \in E(G)} x^{d_s + d_k}$$

By using Table 2.3 in the above equation we get

$$= (m^2n)x^{2+m} + (2m^2+2m))x^{2+m+1} + (mn-2m)(m+2)x^{2+m+2}.$$

After simplifying we have our result:

$$ZG_1(G,x) = (m^2n)x^{m+2} + (2m^2+2m))x^{m+3} + (m^2n + 2mn - 2m^2 - 4m)x^{m+4}. \blacksquare$$

Theorem 2.2.9. Let G be a subdivided m^k-graph of comb graph for k=1. Then second Zagreb polynomial is computed as:

$$ZG_2(G,x) = ((m^2n)x^{2m} + (2m^2+2m))x^{2m+2} + (m^2n + 2mn - 2m^2 - 4m)x^{2m+4}.$$

Proof: SZ polynomial is

$$ZG_2(G,x) = \sum_{sk \in E(G)} x^{d_s d_k}$$

By using Table 2.3 in the given equation we obtain

$$= (m^2n)x^{(2)(m)} + (2m^2+2m))x^{(2)(m+1)} + (mn-2m)(m+2)x^{(2)(m+2)}.$$

After simplifying we have our result

$$ZG_2(G,x) = ((m^2n)x^{2m} + (2m^2+2m))x^{2m+2} + (m^2n + 2mn - 2m^2 - 4m)x^{2m+4}. \blacksquare$$

Theorem 2.2.10. Let G be a subdivided m^k-graph of comb graph for k=1. Then ABC-index is computed as:

$$ABC(G) = \frac{1}{\sqrt{2}}(2m^2n + 2mn - 2m).$$

Proof: ABC-index is

$$ABC(G) = \sum_{pq \in E(G)} \sqrt{\frac{d_p + d_q - 2}{d_p \times d_q}}$$

By using Table 2.3 in the above equation we obtain

$$= m^2n\sqrt{\frac{2+m-2}{(2)(m)}} + (2m^2+2m)\sqrt{\frac{2+m+1-2}{(2)(m+1)}} + (m^2n + 2mn - 2m^2 - 4m)\sqrt{\frac{2+m+2-2}{(2)(m+2)}}.$$

After simplifying we have our result:

$$ABC(G) = \frac{1}{\sqrt{2}}(2m^2n + 2mn - 2m). \blacksquare$$

Theorem 2.2.11. Let G be a subdivided m^k-graph of comb graph for k=1. Then GA-index is computed as:

$$GA(G) = (m^2n)\frac{2\sqrt{2m}}{2m+2} + (2m^2+2m)\frac{2\sqrt{2m+2}}{m+3} + (m^2n + 2mn - 2m^2 - 4m)\frac{2\sqrt{2m+4}}{m+4}.$$

Proof: GA-index is

$$GA(G) = \sum_{pq \in E(G)} \frac{2\sqrt{d_p \times d_q}}{d_p + d_q}$$

By using Table 2.3 we get

$$= (m^2n)2\sqrt{\frac{(2)(m)}{2+m}} + (2m^2+2m)2\sqrt{\frac{(2)(m+1)}{2+m+1}} + (m^2n + 2mn - 2m^2 - 4m)2\sqrt{\frac{(2)(m+2)}{2+m+2}}$$

After simplifying we get our result

$$GA(G) = (m^2n)\frac{2\sqrt{2m}}{2m+2} + (2m^2+2m)\frac{2\sqrt{2m+2}}{m+3} + (m^2n + 2mn - 2m^2 - 4m)\frac{2\sqrt{2m+4}}{m+4}. \blacksquare$$

Conclusion

In this study we constructed subdivided m^k-graph of path graph, cycle graph, friendship graph, fan graph, comb graph, ladder graph and triangular ladder graph. We also computed some degree based topological indices for subdivided m^k-graph generated by path graph, cycle graph, friendship graph, fan graph, comb graph, ladder graph and triangular ladder graph. Some closed formulas for computing several degree based topological indices of subdivided m^k-graphs were also provided.

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