

FULL PAPER

Computing metric and partition dimension of tessellation of plane by boron nanosheets

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Metric dimension $dim(G)$ and portion dimension $pd(G)$ are usually related as $pd(G) \leq dim(G) + 1$. However, if the partition dimension is significantly smaller than the metric dimension, then it is termed as discrepancy. This paper mainly deals with metric dimension and partition dimension of tessellation of plane by boron nanosheets. It has been proved that there is a discrepancy between the mentioned parameters of the boron nanosheets. Moreover, some induced subgraphs of the stated sheets have been considered for the study of their metric dimension.

KEYWORDS

Metric dimension; partition dimension; basis; resolving set; boron nanotube.

Introduction

Nanotechnology and nano science are the use of significantly small objects and are applicable to other scientific fields. New materials and de-vices are created with the help of nanotechnology with a wide range of applications in different areas such as electronics, computer, and medicines. People are thinking that the nanotechnology will revolutionize the 21st century just like 20th century which is revolutionized by communication and entertainment technology. Different nanotubes are being used in this technology. A nanotube is formed by rolling a plane nanosheet in cylindrical way. Some important nano structures are boron α -nanotubes, boron α - β -naotubes, carbon nanotubes, and boron triangular nanotubes.

Recently, pure boron triangular nanotubes have been discovered and their discovery has challenged the dominance carbon nanotubes.

A boron triangular nanotube was first constructed from triangular sheet in 2004, [2, 17]. If we insert exactly one atom into each hexagon of a hexagonal sheet, then we will have a boron triangular nanotube. The scientists are of the opinion that these tubes are much better than the hexagonal nanotubes [2, 25]. Peter Miller [17] believed that if carbon nanotubes were shining on the horizon in 2007, then it would be possible that the year 2008 was be the time for boron nanotubes to shine. Sohrab Ismail-Beigi [26] predicted that we are heading towards a superconducting nano computer with boron wiring, and after some times this prediction comes true when scientists would be able to make a smallest superconductor by using molecular superconducting boron wires. Special boron sheets have been constructed from the old hexagonal sheets by inserting exactly one atom into the center of some particular hexagons and the said special

sheets are known as boron α -sheet and boron α - β -sheet [15, 25].

A path with minimum length between $u, v \in V(G)$ is the shortest path and the length is the *distance* $d(u, v)$, of G . Let $W = \{w_1, w_2, \dots, w_k\}$ be a set of G whose vertices are arranged in a specific order and let v be a vertex of G . A vector $r(v|W) = (d(v, w_1), d(v, w_2), \dots, d(v, w_k))$ with k components gives the representation of v with respect to W .

If for any vertices $t \neq p \in V(G)$, $r(t|W) \neq r(p|W)$ with respect to W , then W is called a *resolving set or locating set* for G [3]. A set which resolves all elements of $V(G)$ is a basis for G and its cardinality is the *metric* of G , denoted by $dim(G)$. The readers are invited to consider the following articles for detailed study [4, 5, 11, 12, 16, 20, 23, 24, 27, 29-39].

A given arranged set of vertices $W = \{w_1, w_2, \dots, w_k\}$ of a graph G , the i th component of the vector $r(v|W) = 0 \leftrightarrow v = w_i$. Thus, one can easily show that W is a basis set by verifying only that for all $x \neq y \in V(G) \setminus W$, $r(x|W) \neq r(y|W)$

The following lemma proposed us the way of choosing the elements of the basis:

Lemma 1.1. [28] *Let W be a resolving set for a connected graph G and $u, v \in V(G)$. If $d(u, w) = d(v, w)$ for all vertices $w \in V(G) \setminus \{u, v\}$, then $\{u, v\} \cap W \neq \emptyset$.*

If $G_n: \Sigma = (G_n)_{n \geq 1}$ is a family of connected graphs with order $|V(G_n)| = \Omega(n)$ and $\lim_{n \rightarrow \infty} \Omega(n) = \infty$, this family will have metric dimension bounded by a positive constant a i.e. $dim(G_n) \leq a, \forall n \geq 1$, and if no such number exists, then $dim(G_n)$ is unbounded. If $(G_n) = a, \forall n \geq 1$, then $dim(G_n)$ is constant [13].

Let $S \subset V(G)$ and a vertex $v \in V(G)$, the distance between point and a set is defined as $d(v, S) = \min\{d(v, x) : x \in S\}$. A p -partition $\Pi = (S_1, S_2, \dots, S_p)$ of $V(G)$ will give us representation

$r(v|\Pi) = (d(v, S_1), d(v, S_2), \dots, d(v, S_p))$ of v with respect to Π . If all the vectors $r(v|\Pi)$ are distinct for every $v \in V(G)$, then Π is called a *resolving partition*. One such partition with

least cardinality is termed as the *partition dimension* of G , denoted by $pd(G)$. Let $\Pi = (S_1, S_2, \dots, S_p)$ be partition of $V(G)$ arranged in a specific order, and if $u \in S_i, v \in S_j$ such that $1 \leq i, j \leq k$ and $i \neq j$, then $r(u|\Pi) \neq r(v|\Pi)$ since $d(u, S_i) = 0$ but $d(v, S_i) \neq 0$. If the vertices of the same partite set have different representation, then this will guaranty that the partition is resolving.

When $d(u, S_i) \neq d(v, S_i)$ then the class S_i distinguishes vertices u and v .

The following Lemma suggests us the way of choosing the elements of partite sets [7].

Lemma 1.2. [7] *Let Π be a resolving partition of $V(G)$ and $u, v \in V(G)$. If $d(u, w) = d(v, w)$ for all vertices $w \in V(G) \setminus \{u, v\}$, then u and v belong to different classes of Π .*

The researchers are interested in finding the relation between partition dimension and metric dimension. One of the most popular and useful relations between the said parameters is given in [7] as follows: $pd(G) \leq dim(G) + 1$.

However, the partition dimension is significantly smaller than metric dimension, and if this happens for certain families of graphs then it is termed as a discrepancy between these two parameters.

These notions have application in diverse areas of science and technology, for the representation of chemical compounds in chemistry [5, 11, 30], pattern recognition and image processing [16, 14].

For the study of discrepancies between these two distance-based parameters, the readers are suggested to consider these articles [16, 19, 21-24]. These articles motivated us to consider some more graphs generated by tiling of the plane (boron nanotubes) to check the discrepancy between these parameters.

A carbon nanosheet is a sheet on plane that is generated by hexagons with n rows and m columns. If we roll up these sheets and merge the last column vertices by corresponding first column vertices, then a

nanotube of order $n \times m$ is formed. A boron α -nanotube and α - β -nanotube can be constructed easily by inserting a single vertex into the center of some particular hexagons and if an atom is inserted into each hexagon of hexagonal tube then the resulting tube is termed as triangular nanotube. These nanotubes are of order $n \times m$.

Some of the important components of the graph are vertices and edges which help to understand the properties of graph in a better way by labeling them.

Since past decades, the researchers have been highly motivated to consider the nano structures for their studies and investigated many graph theoretic properties, some of which are listed as topological descriptors, bipartite edge frustration, group symmetries and chromatic polynomials [1, 9, 10].

Recently, some distance-based parameters have been studied in [19, 21, 22].

All the aforementioned properties of nano structures motivated us to consider boron nano structures to study two fundamental and widely applicable parameters of the graphs termed as metric dimension and partition dimension.

Results and discussion

This paper delved into two highly applicable parameters namely partition dimension and metric dimension of newly defined important structures termed as boron α -sheet and boron α - β -sheet. These structures have infinite elements in their metric basis but finite elements in their resolving partition set.

Some induced subgraphs have also been considered in relation to studying these structures and their metric basis. We found that some of them have fixed metric basis while the other have variable metric basis that vary with change of the order of the structures.

Following lemma shows that metric basis of infinite boron α -sheet and boron α - β -sheet has infinite number of elements.

Lemma 2.1. *The infinite boron α -sheet and boron α - β -sheet have infinite metric basis, i.e., $\dim(\text{boron } \alpha\text{-sheet}) = \dim(\text{boron } \alpha\text{-}\beta\text{-sheet}) = \infty$.*

Proof. Figure 1 represents boron α -sheet showing two vertices labeled by x, y and some vertices z in this graphs with the condition $d(x, z) = d(y, z)$. Suppose there is finite number of elements in the metric basis β of boron α - β -sheet. There are two vertices x, y and a subset A that contains all such elements of graphs which satisfies the condition, $d(x, z) = d(y, z) \leq l$, where $l \in \mathbb{N}$, a set of natural numbers such that $B \subset A$. This leads to the conclusion that $d(x, z) = d(y, z)$ for all $z \in B$, which produces a contradiction. The proof for second boron sheet can be established in similar fashion.

Following lemma is providing good bounds of resolving partition of boron α -sheet and boron α - β -sheet.

Lemma 2.2. *We have $pd(\text{boron } \alpha\text{-sheet}) = 3$ and $3 \leq pd(\text{boron } \alpha\text{-}\beta\text{-sheet}) \leq 4$.*

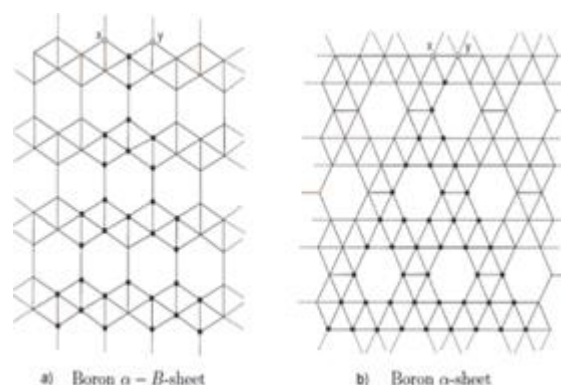


FIGURE1 Bold vertices have same distances from x and y

Proof. A graph G has only elements in its resolving partition if it is a path graph and this result was proved by Chartr and et al. [6]. This result holds true even for infinite path. This shows that $pd(\text{boron } \alpha\text{-sheet})$ and $pd(\text{boron } \alpha\text{-}\beta\text{-sheet})$ is bounded below by 3.s.

A resolving partition comprising three elements of boron α -sheet and a resolving partition comprising four elements of Boron α - β -are given in Figure 2. This shows that

$pd(\text{boron } \alpha\text{-sheet})=3$ and $pd(\text{boron } \alpha\text{-}\beta\text{-sheet})$ is bounded below by 3 and bounded above by 4.

If every face of a finite connected edge crossing free graph is bounded by $4k$ -cycle having unit length, then that graph is termed as k -polyomino system or one can say that it is the union of edge connected $4k$ -cycles.

There are many induced subgraphs of the stated boron nano sheets, some of which are given in Figure 3. An induced subgraph AA_n is the union of 6-cycles whose edges are connected. Also, AA'_n is the pair wise zig-zag chain induced by C_3 with $2n$ pairs; the graph AA''_n is defined as an edge connected union of C_6 and $(n-1)P_3$. Figure 4 represents an induced subgraph XX_n defined as an edge connected union n groups one hexagon and two W_6 (wheel of length 7) alternately. Figure 5 represents a polyomino chain of 6-cycles and is denoted by GG_n . Note that these induced subgraphs are of order k .

In the next theorem, the metric dimension of some afore mentioned induced subgraphs has been studied and proved that there exist induced subgraphs of these boron nano sheets, some of which have metric dimension depending upon n and others have a constant metric dimension.

Theorem 2.1. i) For every integer $n \geq 1$, $\dim(AA_n)=2$.

ii) For every integer $n \geq 2$, $\dim(AA'_n)=\dim(AA''_n)=n$.

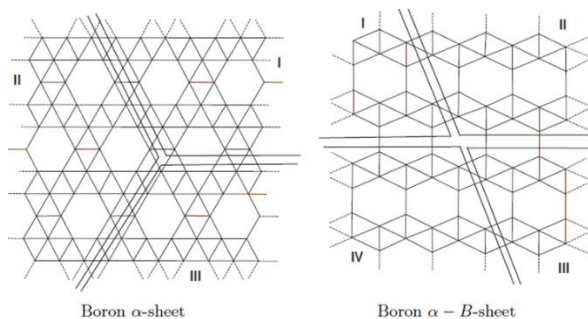


FIGURE 2 A boron α -sheet with 3-partition and a boron α - β -sheet with 4-partition.

Proof. i) The metric bases of induced subgraph AA_n will contain more than two elements (vertices) because a connected graph G has metric basis containing only one element if and only if it is P_n as proved in [5].

In AA_n , u_i and v_i , where $1 \leq i \leq k/2$ are representing the vertices of C_6 that are on the upper half and lower half of C_6 , respectively as shown in Figure 3. For $\dim(AA_n) \leq 2$, one has to verify that $W=\{v_1, u_2\}$ resolves $V(AA_n)$ and to do this the unique representation of all the elements of $V(AA_n) \setminus W$ with respect to W are required. They are given below

$$r(u_i | W) = (1, 1), r(v_2 | W) = (3, 1),$$

$$r(v_i | W) = (i-1, i-1), \text{ for } 3 \leq i \leq \frac{k}{2}.$$

$$\text{And } r(u_i | W) = (i-2, i), \text{ for } 3 \leq i \leq \frac{k}{2}.$$

From the discussions given above we have $\dim(AA_n)=2$.

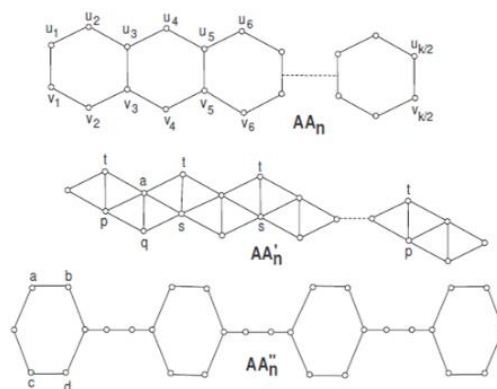


FIGURE 3 Some induced subgraphs of boron sheets

In D_n , v_j and u_j , $1 \leq j \leq k/2$ represents the vertices of C_8 or C_6 that lies on upper and lower half, respectively as shown in Figure 3. For $\dim(D_n) \leq 2$, it suffices to prove that the set $W=\{v_1, u_2\}$ resolves $V(D_n)$. So for this, unique represents for all elements of $V(D_n) \setminus W_1$ with respect to W_1 are required i.e.

$$r(v_2 | W_1) = (1, 3), r(u_1 | W_1) = (1, 1)$$

$$r(v_j | W_1) = (j-1, j-1), \text{ for } 3 \leq j \leq \frac{k}{2}$$

$$\text{And } r(u_j | W_1) = (j, j-2), \text{ for } 3 \leq j \leq \frac{k}{2}.$$

All the above discussions lead us to conclude that $\dim(D_n)=2$.

ii) In AA'_n , there exists two vertices t and p that have the same distances to all rest of the vertices of AA'_n other than q as shown in Figure 3. So the vertex q is distinguishing the vertices t and p that are in the first pair of C_3 from left to right. If q is not part of any resolving set with least number of elements of AA'_n . This clearly indicates that any minimum resolving set of AA'_n must contain at least one of t, p . In this manner, a resolving set with least number of elements can be formed by selecting exactly one third degree vertex of kind t from every quadruple of C_3 of AA'_n (in at least $2n$ ways) and this will form a metric basis, leading to prove the dimension of this subgraph.

The result can be followed by stating that there are $n!$ ways to arrange these vertices. The metric basis for the graph AA'_n can be formed in similar manners by selecting at least one of the vertices a, b, c and d . Moreover, there are $n!4n$ minimum resolving sets that lead to possibly no metric basis.

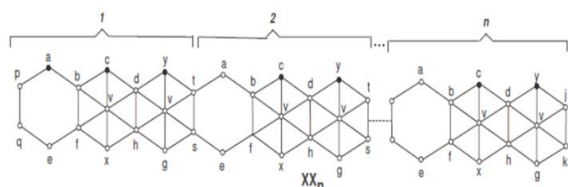


FIGURE 4 The induced subgraph XX_n of boron α -sheet

Theorem 2.2. For every $n \geq 1$, $\dim(XX_n) = 2n + 1$.

Proof. The graph XX_n has divided into groups comprising three consecutive hexagons and each group is labeled by numbers 1 to n from left to right as shown in Fig. 4. One can easily verify that $\dim(XX_1) = 3$ with minimum resolving set $\{a, b, y\}$. Now $\dim(XX_n) = 2n + 1$ for $n \geq 2$ can easily be proven by double inequality.

The upper bound can be achieved by forming a resolving with $2n + 1$ number of

elements in it. This can be constructed in the following fashion:

First of all, consider the first group of three hexagons labeled as 1 and select a, c and y type vertices and then select only two c and y type vertices from each group of hexagons labeled by 2 to n from left to right.

If any pair of vertices of XX_n has the same distance to some vertex other H , then this pair can be distinguished by some of the remaining vertices of H . This argument helps to show that $\dim(XX_n) \leq 2n + 1$, for $n \geq 1$. To prove that $\dim(XX_n) \geq 2n + 1$ for $n \geq 1$, one can verify that every group of three hexagons labeled by 2 to n contributes three vertices to the resolving set of XX_n while the first group labeled by 1 contributes only two vertices. There are two pair of vertices of type c and x , and of type y and g lying in the second and the third hexagons, respectively, of first group of three hexagons labeled by 1 that distinguishes the pair of vertices of type d and h common in second-third hexagons from left to right. Any pair of vertices of the third hexagons can easily be distinguished in the similar fashion instead of the pair of vertices lying in the first hexagon from left and the vertices of type b, f that can only be identified by c and x type vertices lying in the second hexagon from left.

This implies that any minimum resolving set of XX_n contains at least two vertices from every group of three hexagons labeled by 2 to n from left to right.

Now assign a binary variable X_i to i^{th} group of hexagons having value 1 if it contributes a vertex in the resolving set H of XX_n and 0 if does not, one can conclude:

$$X_1 = 3; X_2 + X_3 \geq 4;$$

$$X_3 + X_4 \geq 4;$$

...

$$X_4 + X_5 \geq 4 X_{n-1} + X_n \geq 4$$

After adding above inequalities, we get:

$$S = X_2 + 2X_3 + 2X_4 + \dots + 2X_{n-1} + X_n \geq 4(n-2).$$

Hence we get

$$2|H| = 2 \sum_{i=1}^n X_i = S + X_2 + X_n + 6 \geq 4n - 8 + 10.$$

This shows that $|H| \geq 2n+1$, $\dim(XX_n) \geq 2n+1$, and the proof is complete.

Theorem 2.3. For every $n \geq 2$, $\dim(GG_n) = n+1$.

Proof. The graph GG_n has divided into pair of hexagons labeled as 1,2,3,...,n from left to right as shown in Figure 5. One can easily verify that $\dim(GG_1) = 2$ with minimum resolving set $\{b,d\}$. Now $\dim(GG_n) = n+1$, for $n \geq 2$ can be proven by double inequality.



FIGURE 5 The induced subgraph GG_n of boron sheet

The upper bound can be achieved by forming a resolving B with $n+1$ number of elements in it. This can be constructed in the following fashion:

- For n even, select vertices from pair of hexagons labeled as 1 of type b and d , from every pair of the upper hexagons labeled as 2,4,...,n select a p type vertex and select a d type vertex from each pair of lower hexagons of GG_n labeled as 3,5,...,n-1 from left to right.
- For n odd, select a pair of vertices of types b and d from hexagon labeled by 1 and a p type vertex from the pair of upper hexagons labeled as 2,4,...,n-1 in GG_n and a d type vertex from pair of lower hexagons labeled as 3,5,...,n from left to right in GG_n . If any pair of vertices of GG_n has the same distance to some vertex B , then this pair can be distinguished by some of the remaining vertices of B . This argument helps to shows that $\dim(GG_n) \leq n+1$, for $n \geq 1$. Now $\dim(GG_n) \geq n+1$, for $n \geq 1$ one can easily verify that the first hexagon of GG_n contributes at least two vertices to every resolving set and every pair of hexagons

labeled as 2,3,...,n from left to right in the pair wise zig-zag will contribute at least one element to every resolving set of GG_n . The pair of vertices of type b and d distinguishes all vertices of pair of hexagons labeled by 1 from left to right except a p type vertex lying in the third hexagon. The pair of vertices of type p and s lying in the third hexagon and the pair vertices of type v and u lying in fourth hexagon of GG_n distinguishes the pair of vertices of type q and y lying in third hexagon from left to right. Therefore, one of them must be part of the minimum resolving set and one can easily establish the argument in the similar fashion for the remaining vertex pairs lying in the third and fourth hexagon. This shows that every pair of consecutive hexagons of GG_n must contribute at least one vertex in any resolving set with minimum number of elements. Now assign a binary variable g_i to i^{th} hexagon having value 1 if it contributes a vertex in the resolving set B of GG_n and 0 if does not, one can conclude:

$$g_1 = 2; g_2 + g_3 \geq 1;$$

$$g_3 + g_4 \geq 1;$$

$$g_4 + g_5 \geq 1$$

...

$$g_{n-1} + g_n \geq 1.$$

After adding the above inequalities, we get:

$$S = g_2 + 2g_3 + 2g_4 + \dots + 2g_{n-1} + g_n \geq 2.$$

Hence

$$2|B| = 2 \sum_{i=1}^n g_i = S + g_2 + g_n + 2 \geq n - 2 + 1 + 2$$

This shows that $|B| \geq n+1$, $\dim(GG_n) \geq n+1$ and the proof is complete. ■

Conclusion

In this paper, the existence of discrepancy between $pd(G)$ and $\dim(G)$ has been proved. It has also been proved that there exist some subgraphs of the mentioned sheets with constant $\dim(G)$ while the other with unbounded $\dim(G)$.

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