Received: 01 September 2020 | Revised: 15 October 2020 | Accepted: Revised: 16 October 2020
DOI: 10.22034/ecc.2020.251925.1082

Eurasian Chemical Communications
SAMI

FULL PAPER

Some topological descriptors and algebraic polynomials of $P_m+_FP_m$

Abdul Qudair Baig[a,*] | Adnan Amin[a] | Mohammad Reza Farahani[b] | Muhammad Imran[c] | Murat Cancan[d] | Mehmet Serif Aldemir[e]

[a]*Department of Mathematics, COMSATS Institute of Information Technology, Attock, Pakistan*

[b]*Department of Mathematics, Iran University of Science and Technology (IUST) Narmak, 16844, Tehran, Iran*

[c]*Department of Mathematical Sciences, United Arab Emirates University, P.O. Box 15551, Al Ain, United Arab Emirates*

[d]*Faculty of Education, Van Yuzuncu Yıl University, Zeve Campus, Tuşba, 65080, Van, Turkey*

[e]*Faculty of Science, Van Yuzuncu Yıl University, Zeve Campus, Tuşba, 65080, Van, Turkey*

***Corresponding Author:**
Abdul Qudair Baig
Email: aqbaig1@gmail.com
Tel.:+92-333-4103140

A topological index of G is a quantity related to G that characterizes its topology. Properties of the chemical compounds and topological invariants are related to each other. In this paper, we derive the algebraic polynomials including first and second Zagreb polynomials, and forgotten polynomial for $P_{m+F}P_m$. Further, we worked on the hyper-Zagreb, first and second multiple Zagreb indices, and forgotten index of these graphs. Consider the molecular graph with atoms to be taken as vertices and bonds can be shown by edges. For such graphs, we can determine the topological descriptors showing their bioactivity as well as their physiochemical characteristics. Moreover, we derive graphical representation of our outcomes, depicting the technical dependence of topological indices and polynomials on the involved structural parameters.

KEYWORDS

Algebraic polynomial; topological descriptor; Zagreb indices.

Introduction

In chemical graph theory, and Mathematics Chemistry, a topological descriptor is a sort of a molecular topological invariant that is computed based on the molecular structure of a chemical compound. A big quantity of chemical experiments needs a resolution of the chemical characteristics of compounds and drugs. The chemical-based experiments demonstrate that there is strong inherent correlation between the chemical characteristics of chemical compounds and drugs, and molecular structures. Topological invariants deliberated for the chemical structures can be helpful for us to work on the physical features, chemical reactivity, and biological activity.

A topological index is designed by reforming a chemical structure into a quantity. These topological invariants are associated with some physicochemical characteristics like stability, boiling point, strain energy etc. of chemical compounds. These are computed by the help of their definitions. Chemical reaction network theory is a field of applied mathematics that is beneficial to model the changing in real world chemical systems. Since its beginning in the 1960s, it has attracted the attraction of the researchers in developing research areas, only because of its importance in biochemistry and theoretical Chem. It has also drawn the interest among pure mathematicians because of its interesting problems that come to light from the Mathematics patterns in structures of material.

Shirdel et al. [1] worked on the hyper Zagreb index and gave its mathematics representation as follows:

$$HM(G)=\sum_{uv\in E(G)}(d_u+d_v)^2 \qquad (1)$$

Ghorbani and Azimi [2] worked on two new variations of Zagreb indices; first multiple Zagreb index

$$PM_1(G) = \prod_{uv \in E(G)} (d_u + d_v) \quad (2)$$

and second multiple Zagreb index

$$PM_2(G) = \prod_{uv \in E(G)} (d_u \times d_v) \quad (3)$$

Furtula and Gutman [3] proposed a very beneficial topological index known as the forgotten index and described as:

$$F(G) = \sum_{uv \in E(G)} ((d_u)^2 + (d_v)^2) \quad (4)$$

The first Zagreb polynomial of G is defined in [4, 5] as follows:

$$M_1(G,x) = \sum_{uv \in E(G)} x^{(d_u + d_v)} \quad (5)$$

The second Zagreb polynomial of G is defined in [4, 5] as follows:

$$M_2(G,x) = \sum_{uv \in E(G)} x^{(d_u \times d_v)} \quad (6)$$

$$F(G,x) = \sum_{uv \in E(G)} x^{((d_u)^2 + (d_v)^2)} \quad (7)$$

Main results

In this paper we computed the first Zagreb polynomial, second Zagreb polynomial and forgotten polynomial of $P_m +_F P_m$. We also computed some degree-based topological indices such as first multiple Zagreb index, second multiple Zagreb index, Hyper Zagreb index and forgotten index or F-index of these networks. Further information on topological indices are available in the literature [6-24].

TABLE 1 Edge partition of $P_m +_Q P_m$ based on degree of end vertices of each edge

$(d_u; d_v)$	(2; 3)	(3; 3)	(3; 4)	(4; 4)
Frequency	8	6+4t	10(t-1)t	$4t^2$+5t-3

Q-Operation for P_m

The graph $P_m +_Q P_m$ have $2t^2+7t+6$ vertices and $4t^2+19t+21$ edges (Table 1).

Theorem 1. Let $G \cong P_m +_Q P_m$ be the graph then the first Zagreb polynomial for this graph is:

$$M_1(P_m +_Q P_m, x) = 8x^5 + (6+4t)x^6 + 10(t+1)x^7 + (4t^2 + 5t - 3)x^8.$$

Proof. By definition of first Zagreb polynomial

$$M_1(G,x) = \sum_{uv \in E(G)} x^{(d_u + d_v)} = M_1(P_m +_Q P_m, x) = \sum_{uv \in E_1(P_m +_Q P_m)} x^{(d_u + d_v)}$$

$$+ \sum_{uv \in E_2(P_m +_Q P_m)} x^{(d_u + d_v)} + \sum_{uv \in E_3(P_m +_Q P_m)} x^{(d_u + d_v)} + \sum_{uv \in E_4(P_m +_Q P_m)} x^{(d_u + d_v)}$$

$$= |E_1(P_m +_Q P_m)| x^{(2+3)} + |E_2(P_m +_Q P_m)| x^{(3+3)}$$

$$+ |E_3(P_m +_Q P_m)| x^{(3+4)} + |E_4(P_m +_Q P_m)| x^{(4+4)}$$

$$= 8x^5 + (6+4t)x^6 + 10(t+1)x^7 + (4t^2 + 5t - 3)x^8$$

Theorem 2. Let $G \cong P_m +_Q P_m$ be the graph then the first Zagreb polynomial, second Zagreb polynomial and forgotten polynomial for this graph is

$$M_2(P_m +_Q P_m, x) = 8x^6 + (6+4t)x^9 + 10(t+1)x^{12} + (4t^2 + 5t - 3)x^{16}.$$

Proof. By definition of second Zagreb polynomial we have,

$$M_1(G,x) = \sum_{uv \in E(G)} x^{(d_u \times d_v)}$$

$$M_1(P_m +_Q P_m, x) = \sum_{uv \in E_1(P_m +_Q P_m)} x^{(d_u \times d_v)} + \sum_{uv \in E_2(P_m +_Q P_m)} x^{(d_u \times d_v)}$$

$$+ \sum_{uv \in E_3(P_m +_Q P_m)} x^{(d_u \times d_v)} + \sum_{uv \in E_4(P_m +_Q P_m)} x^{(d_u \times d_v)}$$

$$= |E_1(P_m +_Q P_m)| x^{(2\times 3)} + |E_2(P_m +_Q P_m)| x^{(3\times 3)}$$

$$+ |E_3(P_m +_Q P_m)| x^{(3\times 4)} + |E_4(P_m +_Q P_m)| x^{(4\times 4)}$$

$$= 8x^6 + (6+4t)x^9 + 10(t+1)x^{12} + (4t^2 + 5t - 3)x^{16}$$

Theorem 3. Let $G \cong P_m +_Q P_m$ be the graph then the first Zagreb polynomial, second Zagreb polynomial and forgotten polynomial for this graph is

$$F(P_m +_Q P_m, x) = 8x^{13} + (6+4t)x^{18} + 10(t+1)x^{25} + (4t^2 + 5t - 3)x^{32}.$$

Proof. By definition of forgotten polynomial, we have:

$$F(G,x) = \sum_{uv \in E(G)} x^{((d_u)^2 + (d_v)^2)} = F(P_m +_Q P_m, x)$$

$$= \sum_{uv \in E_1(P_m +_Q P_m)} x^{((d_u)^2 + (d_v)^2)} + \sum_{uv \in E_2(P_m +_Q P_m)} x^{((d_u)^2 + (d_v)^2)}$$

$$+ \sum_{uv \in E_3(P_m +_Q P_m)} x^{((d_u)^2 + (d_v)^2)} + \sum_{uv \in E_4(P_m +_Q P_m)} x^{((d_u)^2 + (d_v)^2)}$$

$$F(P_m +_Q P_m, x) = 8x^{13} + (6+4t)x^{18} + 10(t+1)x^{25} + (4t^2 + 5t - 3)x^{32}$$

Example: Graphs of first Zagreb polynomial, second Zagreb polynomial and forgot-ten polynomial are shown in Figure 1.

Preposition: Let $G \cong P_m +_Q P_m$ be the graph then the hyper Zagreb index, first multiple Zagreb index, second multiple Zagreb index and forgotten index are:

$$HM(P_m +_Q P_m) = 200 + 36(6+4t) + 490(t+1) + 64(4t^2+5t-3);$$

$$PM_1(P_m +_Q P_m) = 5^8 + 6^{6+4t} + 7^{10t+10} + 8^{4t^2+5t-3};$$

$$PM_2(P_m +_Q P_m) = 6^8 + 9^{6+4t} + 12^{10t+10} + 16^{4t^2+5t-3};$$

$$F(P_m +_Q P_m) = 13^8 + 25^{6+4t} + 7^{10t+10} + 32^{4t^2+5t-3}.$$

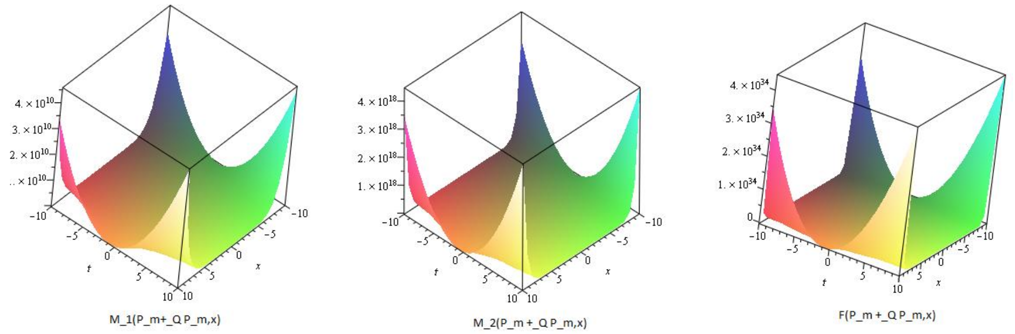

FIGURE 1 Graph of Algebraic polynomials for (P_m+$_Q$$P_m$)

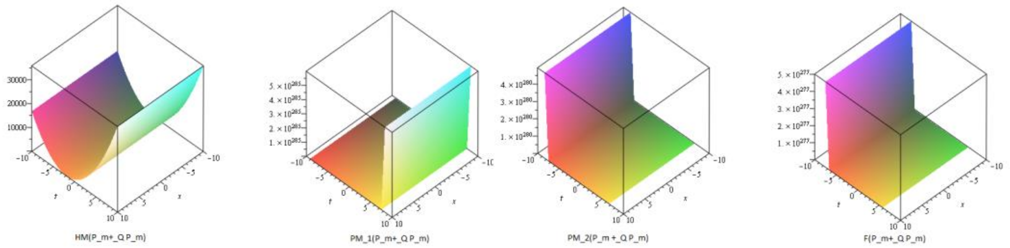

FIGURE 2 Graph of topological indices for (P_m+$_Q$$P_m$)

Proof. (1). Let $G \cong P_m +_Q P_m$ be the graph then by equation (1).

$$HM(G) = \sum_{uv \in E(G)} (d_u + d_v)^2;$$

$$HM(P_m +_Q P_m) = \sum_{uv \in E_1(P_m +_Q P_m)} (d_u + d_v)^2 + \sum_{uv \in E_2(P_m +_Q P_m)} (d_u + d_v)^2$$

$$+ \sum_{uv \in E_3(P_m +_Q P_m)} (d_u + d_v)^2 + \sum_{uv \in E_4(P_m +_Q P_m)} (d_u + d_v)^2;$$

$$HM(P_m +_Q P_m) = |E_1(P_m +_Q P_m)|(5)^2 + |E_2(P_m +_Q P_m)|(6)^2$$

$$+ |E_3(P_m +_Q P_m)|(7)^2 + |E_4(P_m +_Q P_m)|(8)^2;$$

$$HM(P_m +_Q P_m) = 200 + 36(6+4t) + 490(t+1) + 64(4t^2+5t-3).$$

(2). by definition

$$PM_1(G) = \prod_{uv \in E(G)} (d_u + d_v)$$

$$PM_1(P_m +_Q P_m) = \prod_{uv \in E_1(P_m +_Q P_m)} (d_u + d_v) + \prod_{uv \in E_2(P_m +_Q P_m)} (d_u + d_v)$$

$$+ \prod_{uv \in E_3(P_m +_Q P_m)} (d_u + d_v) + \prod_{uv \in E_4(P_m +_Q P_m)} (d_u + d_v)$$

$$= (5)^{|E_1(P_m +_Q P_m)|} + (6)^{|E_2(P_m +_Q P_m)|} + (7)^{|E_3(P_m +_Q P_m)|} + (8)^{|E_4(P_m +_Q P_m)|}$$

$$= (5)^8 + (6)^{(6+4t)} + (7)^{10t+10} + (8)^{(4t^2+5t-3)}.$$

(3). by definition

$$PM_2(G) = \prod_{uv \in E(G)} (d_u \times d_v)$$

$$PM_1(P_m +_Q P_m) = \prod_{uv \in E_1(P_m +_Q P_m)} (d_u + \times) + \prod_{uv \in E_2(P_m +_Q P_m)} (d_u \times d_v)$$

$$+ \prod_{uv \in E_3(P_m +_Q P_m)} (d_u \times d_v) + \prod_{uv \in E_4(P_m +_Q P_m)} (d_u \times d_v)$$

$$= (6)^{|E_1(P_m +_Q P_m)|} + (9)^{|E_2(P_m +_Q P_m)|} + (12)^{|E_3(P_m +_Q P_m)|} + (16)^{|E_4(P_m +_Q P_m)|}$$

$$= (6)^8 + (9)^{(6+4t)} + (12)^{10t+10} + (16)^{(4t^2+5t-3)}.$$

$$F(P_m +_Q P_m) = \prod_{uv \in E(P_m +_Q P_m)} (d_u^2 + d_v^2)$$

(4). $F(P_m +_Q P_m) = \prod_{uv \in E_1(P_m +_Q P_m)} (2^2 + 3^2) + \prod_{uv \in E_2(P_m +_Q P_m)} (3^2 + 3^2)$ ■

$$+ \prod_{uv \in E_3(P_m +_Q P_m)} (3^2 + 4^2) + \prod_{uv \in E_4(P_m +_Q P_m)} (4^2 + 4^2)$$

$$= 13^8 + 18^{(6+4t)} + 13^{10t+10} + 13^{(4t^2+5t-3)}.$$

3D plot for hyper Zagreb index, first Zagreb index, second Zagreb index and forgotten index are shown in Figure 2.

R-Operation for P_m

For graph $P_m +_R P_m$, the cardinality of vertex and edge sets are $2t^2+7t+6$ and $4(t+1)(t+2)$, respectively.

TABLE 2 Edge partition of $G\cong P_m+_R P_m$ based on degree of end vertices of each edge

(d_u; d_v)	(2; 3)	(2; 4)	(2; 5)	(2; 6)	(3; 4)	(3; 5)
Frequency	4	2t	4t	2t²	4	4
(d_u; d_v)	(4; 4)	(4; 6)	(5; 5)	(5; 6)	(6; 6)	.
Frequency	2(t-1)	2t	2(t-1)	2t	2t(t-1)	.

Theorem 4. Let $G\cong P_m+_R P_m$ be the graph then the first Zagreb polynomial for this graph is

$$M_1(P_m+_R P_m,x)=4x^5+2tx^6+4tx^7+2t^2x^8+4x^7+4x^8+(2t-2)x^8+2tx^{10}+(2t-2)x^{10}+2tx^{11}+(2t^2-2t)x^{12}.$$

Proof. By definition of first Zagreb polynomial we have:

$$M_1(G,x)=\sum_{uv\in E(G)}x^{(d_u+d_v)}$$

$$M_1(P_m+_R P_m,x)=\sum_{uv\in E_1(P_m+_R P_m)}x^{(d_u+d_v)}+\sum_{uv\in E_2(P_m+_R P_m)}x^{(d_u+d_v)}+\sum_{uv\in E_3(P_m+_R P_m)}x^{(d_u+d_v)}+\sum_{uv\in E_4(P_m+_R P_m)}x^{(d_u+d_v)}+\sum_{uv\in E_5(P_m+_R P_m)}x^{(d_u+d_v)}$$
$$+\sum_{uv\in E_6(P_m+_R P_m)}x^{(d_u+d_v)}+\sum_{uv\in E_7(P_m+_R P_m)}x^{(d_u+d_v)}+\sum_{uv\in E_8(P_m+_R P_m)}x^{(d_u+d_v)}+\sum_{uv\in E_9(P_m+_R P_m)}x^{(d_u+d_v)}+\sum_{uv\in E_{10}(P_m+_R P_m)}x^{(d_u+d_v)}+\sum_{uv\in E_{11}(P_m+_R P_m)}x^{(d_u+d_v)}.$$

$$M_1(P_m+_R P_m,x)=|E_1(P_m+_R P_m)|x^{(2+3)}+|E_2(P_m+_R P_m)|x^{(2+4)}+|E_3(P_m+_R P_m)|x^{(2+5)}$$
$$+|E_4(P_m+_R P_m)|x^{(2+6)}+|E_5(P_m+_R P_m)|x^{(3+4)}+|E_6(P_m+_R P_m)|x^{(3+5)}+|E_7(P_m+_R P_m)|x^{(4+4)}$$
$$+|E_8(P_m+_R P_m)|x^{(4+6)}+|E_9(P_m+_R P_m)|x^{(5+5)}+|E_{10}(P_m+_R P_m)|x^{(5+6)}+|E_{11}(P_m+_R P_m)|x^{(6+6)}$$
$$M_1(P_m+_R P_m,x)=4x^5+2tx^6+4tx^7+2t^2x^8+4x^7+4x^8+(2t-2)x^8+2tx^{10}+(2t-2)x^{10}+2tx^{11}+(2t^2-2t)x^{12}.\ \blacksquare$$

Theorem 5. Let $G\cong P_m+_R P_m$ be the graph then the second Zagreb polynomial for this graph is

$$M_2(P_m+_R P_m,x)=4x^6+2tx^8+4tx^{10}+2t^2x^{12}+4x^{12}+4x^{15}$$
$$+(2t-2)x^{16}+2tx^{24}+(2t-2)x^{25}+2tx^{30}+(2t^2-2t)x^{36}.$$

Proof. By definition of second Zagreb polynomial, we have:

$$M_2(G,x)=\sum_{uv\in E(G)}x^{(d_u\times d_v)}$$

$$M_2(P_m+_R P_m,x)=\sum_{uv\in E_1(P_m+_R P_m)}x^{(d_u\times d_v)}+\sum_{uv\in E_2(P_m+_R P_m)}x^{(d_u\times d_v)}$$
$$+\sum_{uv\in E_3(P_m+_R P_m)}x^{(d_u\times d_v)}+\sum_{uv\in E_4(P_m+_R P_m)}x^{(d_u\times d_v)}+\sum_{uv\in E_5(P_m+_R P_m)}x^{(d_u\times d_v)}$$
$$+\sum_{uv\in E_6(P_m+_R P_m)}x^{(d_u\times d_v)}+\sum_{uv\in E_7(P_m+_R P_m)}x^{(d_u\times d_v)}+\sum_{uv\in E_8(P_m+_R P_m)}x^{(d_u\times d_v)}$$
$$+\sum_{uv\in E_9(P_m+_R P_m)}x^{(d_u\times d_v)}+\sum_{uv\in E_{10}(P_m+_R P_m)}x^{(d_u\times d_v)}+\sum_{uv\in E_{11}(P_m+_R P_m)}x^{(d_u\times d_v)}$$

$$M_2(P_m+_R P_m,x)=|E_1(P_m+_R P_m)|x^{(2\times3)}+|E_2(P_m+_R P_m)|x^{(2\times4)}$$
$$+|E_3(P_m+_R P_m)|x^{(2\times5)}+|E_4(P_m+_R P_m)|x^{(2\times6)}+|E_5(P_m+_R P_m)|x^{(3\times4)}$$
$$+|E_6(P_m+_R P_m)|x^{(3\times5)}+|E_7(P_m+_R P_m)|x^{(4\times4)}+|E_8(P_m+_R P_m)|x^{(4\times6)}$$
$$+|E_9(P_m+_R P_m)|x^{(5\times5)}+|E_{10}(P_m+_R P_m)|x^{(5\times6)}+|E_{11}(P_m+_R P_m)|x^{(6\times6)}$$
$$M_2(P_m+_R P_m,x)=4x^6+2tx^8+4tx^{10}+2t^2x^{12}+4x^{12}+4x^{15}+(2t-2)x^{16}$$
$$+2tx^{24}+(2t-2)x^{25}+2tx^{30}+(2t^2-2t)x^{36}.\square$$

Theorem 6. Let $G\cong P_m+_R P_m$ be the graph then the forgotten polynomial for this graph is

$$F(P_m+_R P_m,x)=4x^{13}+2tx^{20}+4tx^{29}+2t^2x^{40}+4x^{25}+4x^{34}$$
$$+(2t-2)x^{32}+2tx^{52}+(2t-2)x^{50}+2tx^{61}+(2t^2-2t)x^{72}.$$

Proof. By definition of forgotten polynomial, we have:

$$F(P_m+_R P_m,x)=\sum_{uv\in E_1(P_m+_R P_m)}x^{(d_u^2+d_v^2)}+\sum_{uv\in E_2(P_m+_R P_m)}x^{(d_u^2+d_v^2)}$$
$$+\sum_{uv\in E_3(P_m+_R P_m)}x^{(d_u^2+d_v^2)}+\sum_{uv\in E_4(P_m+_R P_m)}x^{(d_u^2+d_v^2)}+\sum_{uv\in E_5(P_m+_R P_m)}x^{(d_u^2+d_v^2)}$$
$$+\sum_{uv\in E_6(P_m+_R P_m)}x^{(d_u^2+d_v^2)}+\sum_{uv\in E_7(P_m+_R P_m)}x^{(d_u^2+d_v^2)}+\sum_{uv\in E_8(P_m+_R P_m)}x^{(d_u^2+d_v^2)}$$
$$+\sum_{uv\in E_9(P_m+_R P_m)}x^{(d_u^2+d_v^2)}+\sum_{uv\in E_{10}(P_m+_R P_m)}x^{(d_u^2+d_v^2)}+\sum_{uv\in E_{11}(P_m+_R P_m)}x^{(d_u^2+d_v^2)}$$

$$F(P_m+_R P_m,x)=|E_1(P_m+_R P_m)|x^{(4+9)}+|E_2(P_m+_R P_m)|x^{(4+16)}$$
$$+|E_3(P_m+_R P_m)|x^{(4+25)}+|E_4(P_m+_R P_m)|x^{(4+36)}+|E_5(P_m+_R P_m)|x^{(9+16)}$$
$$+|E_6(P_m+_R P_m)|x^{(9+25)}+|E_7(P_m+_R P_m)|x^{(16+16)}+|E_8(P_m+_R P_m)|x^{(16+36)}$$
$$+|E_9(P_m+_R P_m)|x^{(25+25)}+|E_{10}(P_m+_R P_m)|x^{(25+36)}+|E_{11}(P_m+_R P_m)|x^{(36+36)}$$
$$F(P_m+_R P_m,x)=4x^{13}+2tx^{20}+4tx^{29}+2t^2x^{40}+4x^{25}+4x^{34}$$
$$+(2t-2)x^{32}+2tx^{52}+(2t-2)x^{50}+2tx^{61}+(2t^2-2t)x^{72}.\ \blacksquare$$

The graph of first Zagreb polynomial, second Zagreb polynomial and Forgotten polynomial for $P_m+_R P_m$ are shown in Figure 3 below,

Preposition: Let $G\cong P_m+_R P_m$ be the graph then the hyper Zagreb index, first multiple Zagreb index, second multiple Zagreb index and forgotten index are:

$HM(P_m +_R P_m) = 552 + 838t + 328(t-1) + 41472t(t-1);$

$PM_1(P_m +_R P_m) = 5^4 + 6^{2t} + 7^{4t} + 8^{2t^2} + 7^4 + 8^4 + 8^{2t-2} + 10^{2t} + 10^{2t-2} + 11^{2t} + 12^{2t^2+2t};$

$PM_2(P_m +_R P_m) = 6^4 + 8^{2t} + 10^{4t} + 12^{2t^2} + 12^4 + 15^4 + 16^{2t-2} + 24^{2t} + 25^{2t-2} + 30^{2t} + 36^{2t^2+2t};$

$F(P_m +_R P_m) = 13^4 + 20^{2t} + 29^{4t} + 40^{2t^2} + 25^4 + 34^4 + 32^{2t-2} + 52^{2t} + 50^{2t-2} + 61^{2t} + 72^{2t^2+2t}.$

Proof. (1). Let $G \cong P_m +_R P_m$ be the graph then by equation (1), we have:

$$HM(G) = \sum_{uv \in E(G)} (d_u + d_v)^2$$

$$\begin{aligned}
HM(P_m +_R P_m) &= \sum_{uv \in E_1(P_m +_R P_m)} (d_u + d_v)^2 + \sum_{uv \in E_2(P_m +_R P_m)} (d_u + d_v)^2 \\
&+ \sum_{uv \in E_3(P_m +_R P_m)} (d_u + d_v)^2 + \sum_{uv \in E_4(P_m +_R P_m)} (d_u + d_v)^2 + \sum_{uv \in E_5(P_m +_R P_m)} (d_u + d_v)^2 \\
&+ \sum_{uv \in E_6(P_m +_R P_m)} (d_u + d_v)^2 + \sum_{uv \in E_7(P_m +_R P_m)} (d_u + d_v)^2 + \sum_{uv \in E_8(P_m +_R P_m)} (d_u + d_v)^2 \\
&+ \sum_{uv \in E_9(P_m +_R P_m)} (d_u + d_v)^2 + \sum_{uv \in E_{10}(P_m +_R P_m)} (d_u + d_v)^2 + \sum_{uv \in E_{11}(P_m +_R P_m)} (d_u + d_v)^2 \\
&= |E_1(P_m +_R P_m)|(5)^2 + |E_2(P_m +_R P_m)|(6)^2 + |E_3(P_m +_R P_m)|(7)^2 \\
&+ |E_4(P_m +_R P_m)|(8)^2 + |E_5(P_m +_R P_m)|(7)^2 + |E_6(P_m +_R P_m)|(8)^2 \\
&+ |E_7(P_m +_R P_m)|(8)^2 + |E_8(P_m +_R P_m)|(10)^2 + |E_9(P_m +_R P_m)|(10)^2 \\
&+ |E_{10}(P_m +_R P_m)|(11)^2 + |E_{11}(P_m +_R P_m)|(12)^2 \\
&= 552 + 838t + 328(t-1) + 41472t(t-1).
\end{aligned}$$

(2). By definition $PM_1(G) = \prod_{uv \in E(G)} (d_u + d_v)$

$$\begin{aligned}
PM_1(P_m +_R P_m) &= \prod_{uv \in E_1(P_m +_R P_m)} (d_u + d_v) + \prod_{uv \in E_2(P_m +_R P_m)} (d_u + d_v) \\
&+ \prod_{uv \in E_3(P_m +_R P_m)} (d_u + d_v) + \prod_{uv \in E_4(P_m +_R P_m)} (d_u + d_v) + \prod_{uv \in E_5(P_m +_R P_m)} (d_u + d_v) \\
&+ \prod_{uv \in E_6(P_m +_R P_m)} (d_u + d_v) + \prod_{uv \in E_7(P_m +_R P_m)} (d_u + d_v) + \prod_{uv \in E_8(P_m +_R P_m)} (d_u + d_v) \\
&+ \prod_{uv \in E_9(P_m +_R P_m)} (d_u + d_v) + \prod_{uv \in E_{10}(P_m +_R P_m)} (d_u + d_v) + \prod_{uv \in E_{11}(P_m +_R P_m)} (d_u + d_v)
\end{aligned}$$

$$\begin{aligned}
PM_1(P_m +_R P_m) &= (5)^{|E_1(P_m +_R P_m)|} + (6)^{|E_2(P_m +_R P_m)|} + (7)^{|E_3(P_m +_R P_m)|} \\
&+ (8)^{|E_4(P_m +_R P_m)|} + (7)^{|E_5(P_m +_R P_m)|} + (8)^{|E_6(P_m +_R P_m)|} + (8)^{|E_7(P_m +_R P_m)|} \\
&+ (10)^{|E_8(P_m +_R P_m)|} + (10)^{|E_9(P_m +_R P_m)|} + (11)^{|E_{10}(P_m +_R P_m)|} + (12)^{|E_{11}(P_m +_R P_m)|} \\
&= 5^4 + 6^{2t} + 7^{4t} + 8^{2t^2} + 7^4 + 8^4 + 8^{2t-2} + 10^{2t} + 10^{2t-2} + 11^{2t} + 12^{2t^2+2t}.
\end{aligned}$$

(3). By definition $PM_2(G) = \prod_{uv \in E(G)} (d_u \times d_v)$

$$\begin{aligned}
PM_1(P_m +_R P_m) &= \prod_{uv \in E_1(P_m +_R P_m)} (d_u + \times) + \prod_{uv \in E_2(P_m +_R P_m)} (d_u \times d_v) \\
&+ \prod_{uv \in E_3(P_m +_R P_m)} (d_u \times d_v) + \prod_{uv \in E_4(P_m +_R P_m)} (d_u \times d_v) + \prod_{uv \in E_5(P_m +_R P_m)} (d_u \times d_v) \\
&+ \prod_{uv \in E_6(P_m +_R P_m)} (d_u \times d_v) + \prod_{uv \in E_7(P_m +_R P_m)} (d_u \times d_v) + \prod_{uv \in E_8(P_m +_R P_m)} (d_u \times d_v) \\
&+ \prod_{uv \in E_9(P_m +_R P_m)} (d_u + \times) + \prod_{uv \in E_1 0(P_m +_R P_m)} (d_u \times d_v) + \prod_{uv \in E_1 1(P_m +_R P_m)} (d_u \times d_v) \\
&= (6)^{|E_1(P_m +_R P_m)|} + (8)^{|E_2(P_m +_R P_m)|} + (10)^{|E_3(P_m +_R P_m)|} + (12)^{|E_4(P_m +_R P_m)|} \\
&+ (12)^{|E_5(P_m +_R P_m)|} + (15)^{|E_6(P_m +_R P_m)|} + (16)^{|E_7(P_m +_R P_m)|} + (24)^{|E_8(P_m +_R P_m)|} \\
&+ (25)^{|E_9(P_m +_R P_m)|} + (30)^{|E_{10}(P_m +_R P_m)|} + (36)^{|E_{11}(P_m +_R P_m)|} \\
&= 6^4 + 8^{2t} + 10^{4t} + 12^{2t^2} + 12^4 + 15^4 + 16^{2t-2} + 24^{2t} + 25^{2t-2} + 30^{2t} + 36^{2t^2+2t}.
\end{aligned}$$

(4). By definition

$$F(P_m +_R P_m) = \prod_{uv \in E(P_m +_R P_m)} (d_u^2 + d_v^2)$$

$$\begin{aligned}
F(P_m +_R P_m) &= \prod_{uv \in E_1(P_m +_R P_m)} (2^2 + 3^2) + \prod_{uv \in E_2(P_m +_R P_m)} (2^2 + 4^2) \\
&+ \prod_{uv \in E_3(P_m +_R P_m)} (2^2 + 5^2) + \prod_{uv \in E_4(P_m +_R P_m)} (4^2 + 4^2) + \prod_{uv \in E_5(P_m +_R P_m)} (4^2 + 6^2) \\
&+ \prod_{uv \in E_6(P_m +_R P_m)} (3^2 + 5^2) + \prod_{uv \in E_7(P_m +_R P_m)} (3^2 + 4^2) + \prod_{uv \in E_8(P_m +_R P_m)} (4^2 + 4^2) \\
&+ \prod_{uv \in E_9(P_m +_R P_m)} (5^2 + 5^2) + \prod_{uv \in E_{10}(P_m +_R P_m)} (5^2 + 6^2) + \prod_{uv \in E_{11}(P_m +_R P_m)} (6^2 + 6^2) \\
&= 13^4 + 20^{2t} + 29^{4t} + 40^{2t^2} + 25^4 + 34^4 + 32^{2t-2} + 52^{2t} + 50^{2t-2} + 61^{2t} + 72^{2t^2+2t}.
\end{aligned}$$

3D plot for hyper Zagreb index, first Zagreb index, second Zagreb index and forgotten index are shown in Figure 4.

S-Operation for P_m

The graph $P_m +_S P_m$ have order $2t^2+7t+6$ and size $3(t+1)(t+2)$.

Theorem 7. Let $G \cong P_m +_S P_m$ be the graph then the first Zagreb polynomial for this graph is $M_1(P_m +_S P_m, x) = 4x^4 + (4+6t)x^5 + (2t^2+2t-2)x^6 + 2tx^7 + t(t-1)x^8$.

Proof. By definition of first Zagreb polynomial, we have:

$$M_1(G,x) = \sum_{uv \in E(G)} x^{(d_u+d_v)} = M_1(P_m +_S P_m, x)$$

$$\begin{aligned}
&= \sum_{uv \in E_1(P_m +_S P_m)} x^{(d_u+d_v)} + \sum_{uv \in E_2(P_m +_S P_m)} x^{(d_u+d_v)} + \sum_{uv \in E_3(P_m +_S P_m)} x^{(d_u+d_v)} \\
&+ \sum_{uv \in E_4(P_m +_S P_m)} x^{(d_u+d_v)} + \sum_{uv \in E_5(P_m +_S P_m)} x^{(d_u+d_v)} + \sum_{uv \in E_6(P_m +_S P_m)} x^{(d_u+d_v)} \\
&= |E_1(P_m +_S P_m)| x^{(2+2)} + |E_2(P_m +_S P_m)| x^{(2+3)} + |E_3(P_m +_S P_m)| x^{(2+4)} \\
&+ |E_4(P_m +_S P_m)| x^{(3+3)} + |E_5(P_m +_S P_m)| x^{(3+4)} + |E_6(P_m +_S P_m)| x^{(4+4)} \\
&= 4x^4 + (4+6t)x^5 + (2t^2+2t-2)x^6 + 2tx^7 + t(t-1)x^8.
\end{aligned}$$

Theorem 8. Let $G \cong P_m +_S P_m$ be the graph then the second Zagreb polynomial for this graph is $M_2(P_m +_R P_m, x) = 4x^4 + (4+6t)x^6 + (2t^2)x^8 + (2t-2)x^9 + 2tx^{12} + t(t-1)x^{16}$.

Proof. By definition of second Zagreb polynomial, we have:

$$M_2(G,x) = \sum_{uv \in E(G)} x^{(d_u \times d_v)} = M_2(P_m +_S P_m, x)$$

$$\begin{aligned}
&= \sum_{uv \in E_1(P_m +_S P_m)} x^{(d_u \times d_v)} + \sum_{uv \in E_2(P_m +_S P_m)} x^{(d_u \times d_v)} + \sum_{uv \in E_3(P_m +_S P_m)} x^{(d_u \times d_v)} \\
&+ \sum_{uv \in E_4(P_m +_S P_m)} x^{(d_u \times d_v)} + \sum_{uv \in E_5(P_m +_S P_m)} x^{(d_u \times d_v)} + \sum_{uv \in E_6(P_m +_S P_m)} x^{(d_u \times d_v)}
\end{aligned}$$

$$\begin{aligned}
M_2(P_m +_S P_m, x) &= |E_1(P_m +_S P_m)| x^{(2\times2)} + |E_2(P_m +_S P_m)| x^{(2\times3)} \\
&+ |E_3(P_m +_S P_m)| x^{(2\times4)} + |E_4(P_m +_S P_m)| x^{(3\times3)} \\
&+ |E_5(P_m +_S P_m)| x^{(3\times4)} + |E_6(P_m +_S P_m)| x^{(4\times4)} \\
&= 4x^4 + (4+6t)x^6 + (2t^2)x^8 + (2t-2)x^9 + 2tx^{12} + t(t-1)x^{16}.+
\end{aligned}$$

TABLE 3 Edge partition of $P_m+_SP_m$ based on degree of end vertices of each edge.

$(d_u; d_v)$	(2; 2)	(2; 3)	(2; 4)	(3; 3)	(3; 4)	(4; 4)
Frequency	4	6+4t	$2t^2$	2t-2	2t	t(t-1)

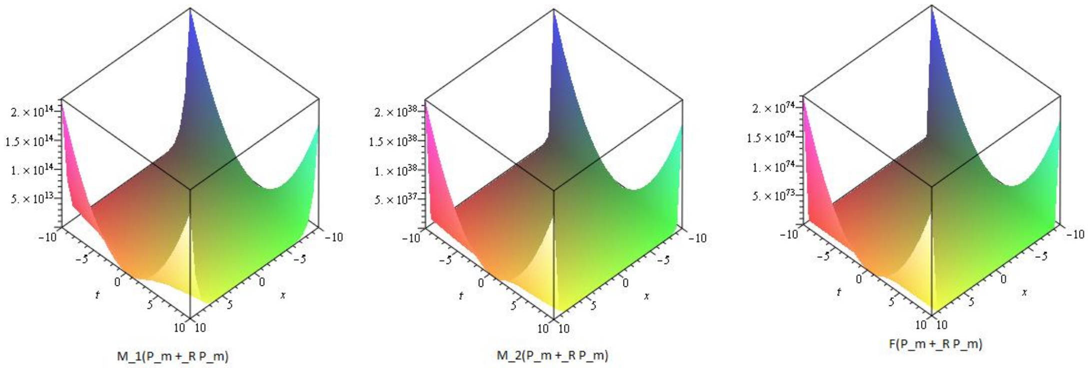

FIGURE 3 Graph of Algebraic polynomials for $P_m+_RP_m$

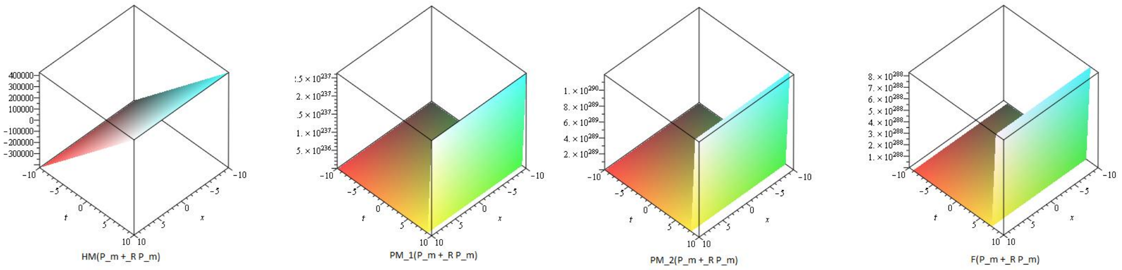

FIGURE 4 Graph of topological indices for $(P_m+_RP_m)$

Theorem 9. Let $G\cong P_m+_SP_m$ be the graph then the forgotten polynomial for this graph is

$$F(P_m +_S P_m, x) = 4x^8 + (4+6t)x^13 + (2t^2)x^20 + (2t-2)x^{18} + 2tx^{25} + t(t-1)x^{32}.$$

Proof. By definition of forgotten polynomial

$$F(G,x) = \sum_{uv\in E(G)} x^{(d_u^2+d_v^2)}$$

$$F(P_m +_S P_m, x) = \sum_{uv\in E_1(P_m+_SP_m)} x^{(d_u^2+d_v^2)} + \sum_{uv\in E_2(P_m+_SP_m)} x^{(d_u^2+d_v^2)} + \sum_{uv\in E_3(P_m+_SP_m)} x^{(d_u^2+d_v^2)}$$
$$+ \sum_{uv\in E_4(P_m+_SP_m)} x^{(d_u^2+d_v^2)} + \sum_{uv\in E_5(P_m+_SP_m)} x^{(d_u^2+d_v^2)} + \sum_{uv\in E_6(P_m+_SP_m)} x^{(d_u^2+d_v^2)}$$
$$= |E_1(P_m +_S P_m)| x^{(4+4)} + |E_2(P_m +_S P_m)| x^{(4+9)} + |E_3(P_m +_S P_m)| x^{(4+16)}$$
$$+ |E_4(P_m +_S P_m)| x^{(9+9)} + |E_5(P_m +_S P_m)| x^{(9+16)} + |E_6(P_m +_S P_m)| x^{(16+16)}$$
$$= 4x^8 + (4+6t)x^13 + (2t^2)x^20 + (2t-2)x^{18} + 2tx^{25} + t(t-1)x^{32}$$

The graph of first Zagreb, second Zagreb and Forgotten polynomials for $P_m+_SP_m$ are shown in Figure 5 below

Proposition: Let $G\cong P_m+_SP_m$ be the graph then the hyper Zagreb index, first multiple Zagreb index, second multiple Zagreb index and forgotten index are

$$HM(P_m +_S P_m) = 64 + 25(4+6t) + 36(2t^2) + 36(2t-2) + (49)2t + 64t(t-1);$$
$$PM_1(P_m +_S P_m) = 4^4 + 5^{4+6t} + 6^{2t^2} + 6^{2t-2} + 7^2 t + 8^t(t-1);$$
$$PM_2(P_m +_S P_m) = 4^4 + 6^{4+6t} + 8^{2t^2} + 9^{2t-2} + 12^2 t + 16^t(t-1);$$
$$F(P_m +_S P_m) = 4^8 + 13^{4+6t} + 20^{2t^2} + 18^{2t-2} + 25^2 t + 32^t(t-1).$$

Proof. (1). Let $G\cong P_m+_SP_m$ be the graph then by equation (1).

$$HM(P_m +_S P_m) = \sum_{uv\in E_1(P_m+_SP_m)} (d_u+d_v)^2 + \sum_{uv\in E_2(P_m+_SP_m)} (d_u+d_v)^2$$
$$+ \sum_{uv\in E_3(P_m+_SP_m)} (d_u+d_v)^2 + \sum_{uv\in E_4(P_m+_SP_m)} (d_u+d_v)^2$$
$$+ \sum_{uv\in E_5(P_m+_SP_m)} (d_u+d_v)^2 + \sum_{uv\in E_6(P_m+_SP_m)} (d_u+d_v)^2$$
$$= |E_1(P_m +_S P_m)|(4)^2 + |E_2(P_m +_S P_m)|(5)^2 + |E_3(P_m +_S P_m)|(6)^2$$
$$+ |E_4(P_m +_S P_m)|(6)^2 + |E_5(P_m +_S P_m)|(7)^2 + |E_6(P_m +_S P_m)|(8)^2$$
$$= 64 + 25(4+6t) + 36(2t^2) + 36(2t-2) + (49)2t + 64t(t-1).$$

(2). By definition

$$PM_1(P_m +_S P_m) = \prod_{uv\in E_1(P_m+_S P_m)} (d_u + d_v) + \prod_{uv\in E_2(P_m+_S P_m)} (d_u + d_v)$$
$$+ \prod_{uv\in E_3(P_m+_S P_m)} (d_u + d_v) + \prod_{uv\in E_4(P_m+_S P_m)} (d_u + d_v)$$
$$+ \prod_{uv\in E_5(P_m+_S P_m)} (d_u + d_v) + \prod_{uv\in E_6(P_m+_S P_m)} (d_u + d_v)$$
$$= (4)^{|E_1(P_m+_S P_m)|} + (5)^{|E_2(P_m+_S P_m)|} + (6)^{|E_3(P_m+_S P_m)|} + (6)^{|E_4(P_m+_S P_m)|}$$
$$+ (7)^{|E_5(P_m+_S P_m)|} + (8)^{|E_6(P_m+_S P_m)|} = 4^4 + 5^{4+6t} + 6^{2t^2} + 6^{2t-2} + 7^2 t + 8^t (t-1).$$

(3). By definition

$$PM_1(P_m +_S P_m) = \prod_{uv\in E_1(P_m+_S P_m)} (d_u + \times) + \prod_{uv\in E_2(P_m+_S P_m)} (d_u \times d_v)$$
$$+ \prod_{uv\in E_3(P_m+_S P_m)} (d_u \times d_v) + \prod_{uv\in E_4(P_m+_S P_m)} (d_u \times d_v)$$
$$+ \prod_{uv\in E_5(P_m+_S P_m)} (d_u + \times) + \prod_{uv\in E_6(P_m+_S P_m)} (d_u \times d_v)$$
$$= (4)^{|E_1(P_m+_S P_m)|} + (6)^{|E_2(P_m+_S P_m)|} + (8)^{|E_3(P_m+_S P_m)|} + (9)^{|E_4(P_m+_S P_m)|}$$
$$+ (12)^{|E_5(P_m+_S P_m)|} + (16)^{|E_6(P_m+_S P_m)|}$$
$$= 4^4 + 6^{4+6t} + 8^{2t^2} + 9^{2t-2} + 12^2 t + 16^t (t-1).$$

(4). By definition

$$F(P_m +_S P_m) = \prod_{uv\in E_1(P_m+_S P_m)} (2^2 + 2^2) + \prod_{uv\in E_2(P_m+_S P_m)} (2^2 + 3^2)$$
$$+ \prod_{uv\in E_3(P_m+_S P_m)} (2^2 + 4^2) + \prod_{uv\in E_4(P_m+_S P_m)} (3^2 + 3^2)$$
$$+ \prod_{uv\in E_5(P_m+_S P_m)} (3^2 + 4^2) + \prod_{uv\in E_6(P_m+_S P_m)} (4^2 + 4^2)$$
$$= 4^8 + 13^{4+6t} + 20^{2t^2} + 18^{2t-2} + 25^2 t + 32^t (t-1).+$$

3D plot for hyper Zagreb index, first Zagreb index, second Zagreb index and forgotten index are shown in Figure 6.

T-Operation for P_m

The graph $P_m+_T P_m$ have $2t^2+7t+6$ vertices and $5t^2+14t+8$ edges.

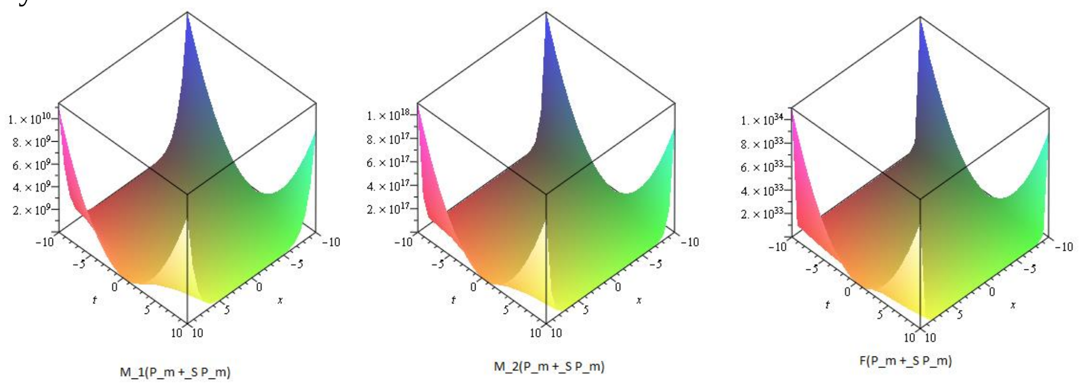

FIGURE 5 Graph of algebraic polynomials for $P_m+_S P_m$

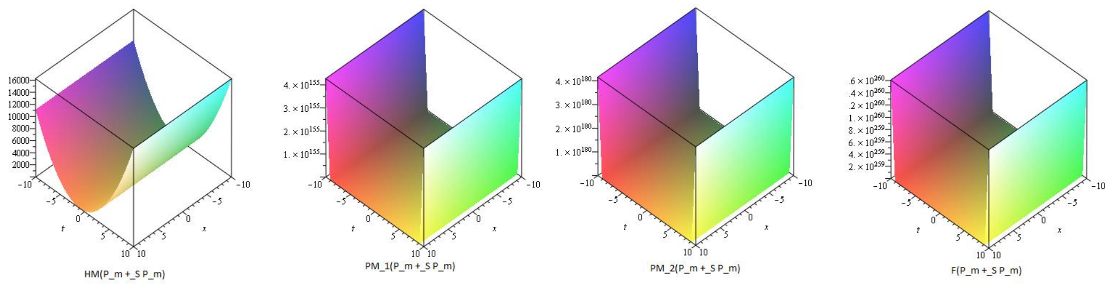

FIGURE 6 Graph of topological indices for ($P_m+_S P_m$)

TABLE 4 Edge partition of $P_m+_T P_m$ based on degree of end vertices of each edge

(d_u; d_v)	(3; 3)	(3; 4)	(3; 5)	(3; 6)	(4; 4)
Frequency	4	8+4t	8	2t	t_2-2t-6
(d_u; d_v)	(4; 5)	(4; 6)	(5; 5)	(5; 6)	(6; 6)
Frequency	4(t-1)	$2t^2$	2(t-1)	2t	2t(t-1)

Theorem 10. Let $G\cong P_m+_TP_m$ be the graph then the first Zagreb polynomial for this graph is

$$M_1(P_m+_TP_m,x)=4x^6+(8+4t)x^7+8x^8+2tx^9+(t^2+2t-6)x^8$$
$$+4(t-1)x^9+(2t^2)x^{1}0+2(t-1)x^{10}+2tx^{11}+2t(t-1)x^{12}.$$

Proof. By definition of first Zagreb polynomial, we have:

$$M_1(G,x)=\sum_{uv\in E(G)}x^{(d_u+d_v)}=M_1(P_m+_TP_m,x)=\sum_{uv\in E_1(P_m+_TP_m)}x^{(d_u+d_v)}$$
$$+\sum_{uv\in E_2(P_m+_TP_m)}x^{(d_u+d_v)}+\sum_{uv\in E_3(P_m+_TP_m)}x^{(d_u+d_v)}+\sum_{uv\in E_4(P_m+_TP_m)}x^{(d_u+d_v)}$$
$$+\sum_{uv\in E_5(P_m+_TP_m)}x^{(d_u+d_v)}+\sum_{uv\in E_6(P_m+_TP_m)}x^{(d_u+d_v)}+\sum_{uv\in E_7(P_m+_TP_m)}x^{(d_u+d_v)}$$
$$+\sum_{uv\in E_8(P_m+_TP_m)}x^{(d_u+d_v)}+\sum_{uv\in E_9(P_m+_TP_m)}x^{(d_u+d_v)}+\sum_{uv\in E_{10}(P_m+_TP_m)}x^{(d_u+d_v)}$$
$$=|E_1(P_m+_TP_m)|x^{(3+3)}+|E_2(P_m+_TP_m)|x^{(3+4)}+|E_3(P_m+_TP_m)|x^{(3+5)}$$
$$+|E_4(P_m+_TP_m)|x^{(3+6)}+|E_5(P_m+_TP_m)|x^{(4+4)}+|E_6(P_m+_TP_m)|x^{(4+5)}$$
$$+|E_7(P_m+_TP_m)|x^{(4+6)}+|E_8(P_m+_TP_m)|x^{(5+5)}+|E_9(P_m+_TP_m)|x^{(5+6)}$$
$$+|E_{10}(P_m+_TP_m)|x^{(6+6)}=4x^6+(8+4t)x^7+8x^8+2tx^9+(t^2+2t-6)x^8$$
$$+4(t-1)x^9+(2t^2)x^{1}0+2(t-1)x^{10}+2tx^{11}+2t(t-1)x^{12}$$

Theorem 11. Let $G\cong P_m+_TP_m$ be the graph then the second Zagreb polynomial for this graph is

$$M_2(P_m+_TP_m,x)=4x^9+(8+4t)x^{12}+8x^{15}+2tx^{18}+(t^2+2t-6)x^{16}$$
$$+4(t-1)x^{20}+(2t^2)x^{24}+2(t-1)x^{25}+2tx^{30}+2t(t-1)x^{36}.$$

Proof. By definition of second Zagreb polynomial, we have:

$$M_2(P_m+_TP_m,x)=\sum_{uv\in E_1(P_m+_TP_m)}x^{(d_u\times d_v)}+\sum_{uv\in E_2(P_m+_TP_m)}x^{(d_u\times d_v)}$$
$$+\sum_{uv\in E_3(P_m+_TP_m)}x^{(d_u\times d_v)}+\sum_{uv\in E_4(P_m+_TP_m)}x^{(d_u\times d_v)}+\sum_{uv\in E_5(P_m+_TP_m)}x^{(d_u\times d_v)}$$
$$+\sum_{uv\in E_6(P_m+_TP_m)}x^{(d_u\times d_v)}+\sum_{uv\in E_7(P_m+_TP_m)}x^{(d_u\times d_v)}+\sum_{uv\in E_8(P_m+_TP_m)}x^{(d_u\times d_v)}$$
$$+\sum_{uv\in E_9(P_m+_TP_m)}x^{(d_u\times d_v)}+\sum_{uv\in E_{10}(P_m+_TP_m)}x^{(d_u\times d_v)}$$
$$=|E_1(P_m+_TP_m)|x^{(2\times3)}+|E_2(P_m+_TP_m)|x^{(2\times4)}+|E_3(P_m+_TP_m)|x^{(2\times5)}$$
$$+|E_4(P_m+_TP_m)|x^{(2\times6)}+|E_5(P_m+_TP_m)|x^{(3\times4)}+|E_6(P_m+_TP_m)|x^{(3\times5)}$$
$$+|E_7(P_m+_TP_m)|x^{(4\times4)}+|E_8(P_m+_TP_m)|x^{(4\times6)}+|E_9(P_m+_TP_m)|x^{(5\times5)}$$
$$+|E_{10}(P_m+_TP_m)|x^{(5\times6)}$$
$$M_2(P_m+_TP_m,x)=4x^9+(8+4t)x^{12}+8x^{15}+2tx^{18}+(t^2+2t-6)x^{16}$$
$$+4(t-1)x^{20}+(2t^2)x^{24}+2(t-1)x^{25}+2tx^{30}+2t(t-1)x^{36}.$$

Theorem 12. Let $G\cong P_m+_TP_m$ be the graph then the forgotten polynomial for this graph is

$$F(P_m+_TP_m,x)=4x^{18}+(8+4t)x^{25}+8x^{34}+2tx^{45}+(t^2+2t-6)x^{32}$$
$$+4(t-1)x^{41}+(2t^2)x^{52}+2(t-1)x^{50}+2tx^{61}+2t(t-1)x^{72}.$$

Proof. By definition of forgotten polynomial, we have:

$$F(G,x)=\sum_{uv\in E(G)}x^{(d_u^2+d_v^2)}$$
$$F(P_m+_TP_m,x)=\sum_{uv\in E_1(P_m+_TP_m)}x^{(d_u^2+d_v^2)}+\sum_{uv\in E_2(P_m+_TP_m)}x^{(d_u^2+d_v^2)}+$$
$$\sum_{uv\in E_3(P_m+_TP_m)}x^{(d_u^2+d_v^2)}+\sum_{uv\in E_4(P_m+_TP_m)}x^{(d_u^2+d_v^2)}+\sum_{uv\in E_5(P_m+_TP_m)}x^{(d_u^2+d_v^2)}$$
$$+\sum_{uv\in E_6(P_m+_TP_m)}x^{(d_u^2+d_v^2)}+\sum_{uv\in E_7(P_m+_TP_m)}x^{(d_u^2+d_v^2)}+\sum_{uv\in E_8(P_m+_TP_m)}x^{(d_u^2+d_v^2)}$$
$$+\sum_{uv\in E_9(P_m+_TP_m)}x^{(d_u^2+d_v^2)}+\sum_{uv\in E_{10}(P_m+_TP_m)}x^{(d_u^2+d_v^2)}$$
$$=|E_1(P_m+_TP_m)|x^{(9+9)}+|E_2(P_m+_TP_m)|x^{(9+16)}+|E_3(P_m+_TP_m)|x^{(9+25)}$$
$$+|E_4(P_m+_TP_m)|x^{(9+36)}+|E_5(P_m+_TP_m)|x^{(16+16)}+|E_6(P_m+_TP_m)|x^{(16+25)}$$
$$+|E_7(P_m+_TP_m)|x^{(16+36)}+|E_8(P_m+_TP_m)|x^{(25+25)}+|E_9(P_m+_TP_m)|x^{(25+36)}$$
$$+|E_{10}(P_m+_TP_m)|x^{(36+36)}$$
$$F(P_m+_TP_m,x)=4x^{18}+(8+4t)x^{25}+8x^{34}+2tx^{45}+(t^2+2t-6)x^{32}$$
$$+4(t-1)x^{41}+(2t^2)x^{52}+2(t-1)x^{50}+2tx^{61}+2t(t-1)x^{72}.$$

The graph of first Zagreb, second Zagreb and Forgotten polynomials for $P_m+_SP_m$ are shown in Figure 7. Below.

Proposition: Let $G\cong P_m+_TP_m$ be the graph then the hyper Zagreb index, first multiple Zagreb index, second multiple Zagreb index and forgotten index are

$$HM(P_m+_TP_m)=656+49(8+4t)+81(2t)+64(2t^2+2t-6)$$
$$+(324)(t-1)+200(t^2)+200(t-1)+242t+288t(t-1);$$
$$PM_1(P_m+_TP_m)=6^4+7^{(8+4t)}+8^8+9^{(2t)}+8^{(2t^2+2t-6)}+(9)^4(t-1)$$
$$+10^{(2t^2)}+10^2(t-1)+11^2t+12^2t(t-1);$$
$$PM_2(P_m+_TP_m)=9^4+12^{(8+4t)}+15^8+18^{(2t)}+16^{(2t^2+2t-6)}$$
$$+(20)^4(t-1)+24^{(2t^2)}+25^2(t-1)+30^2t+36^2t(t-1);$$
$$F(P_m+_TP_m)=18^4+25^{(8+4t)}+34^8+45^{(2t)}+32^{(2t^2+2t-6)}$$
$$+(41)^4(t-1)+52^{(2t^2)}+50^2(t-1)+61^2t+72^2t(t-1).$$

Proof. (1). Let $G\cong P_m+_TP_m$ be the graph then by equation (1), we have:

$$HM(P_m+_TP_m)=\sum_{uv\in E_1(P_m+_TP_m)}(d_u+d_v)^2+\sum_{uv\in E_2(P_m+_TP_m)}(d_u+d_v)^2$$
$$+\sum_{uv\in E_3(P_m+_TP_m)}(d_u+d_v)^2+\sum_{uv\in E_4(P_m+_TP_m)}(d_u+d_v)^2+\sum_{uv\in E_5(P_m+_TP_m)}(d_u+d_v)^2$$
$$+\sum_{uv\in E_6(P_m+_TP_m)}(d_u+d_v)^2+\sum_{uv\in E_7(P_m+_TP_m)}(d_u+d_v)^2+\sum_{uv\in E_8(P_m+_TP_m)}(d_u+d_v)^2$$
$$+\sum_{uv\in E_9(P_m+_TP_m)}(d_u+d_v)^2+\sum_{uv\in E_10(P_m+_TP_m)}(d_u+d_v)^2$$
$$=|E_1(P_m+_TP_m)|(6)^2+|E_2(P_m+_TP_m)|(7)^2+|E_3(P_m+_TP_m)|(8)^2$$
$$+|E_4(P_m+_TP_m)|(9)^2+|E_5(P_m+_TP_m)|(8)^2+|E_6(P_m+_TP_m)|(9)^2$$
$$+|E_7(P_m+_TP_m)|(10)^2+|E_8(P_m+_TP_m)|(10)^2+|E_9(P_m+_TP_m)|(11)^2$$
$$+|E_{10}(P_m+_TP_m)|(12)^2=656+49(8+4t)+81(2t)+64(2t^2+2t-6)$$
$$+(324)(t-1)+200(t^2)+200(t-1)+242t+288t(t-1).$$

(2). By definition

$$PM_1(P_m +_T P_m) = \prod_{uv\in E_1(P_m+_T P_m)} (d_u + d_v) + \prod_{uv\in E_2(P_m+_T P_m)} (d_u + d_v)$$
$$+ \prod_{uv\in E_3(P_m+_T P_m)} (d_u + d_v) + \prod_{uv\in E_4(P_m+_T P_m)} (d_u + d_v) + \prod_{uv\in E_5(P_m+_T P_m)} (d_u + d_v)$$
$$+ \prod_{uv\in E_6(P_m+_T P_m)} (d_u + d_v) + \prod_{uv\in E_7(P_m+_T P_m)} (d_u + d_v) + \prod_{uv\in E_8(P_m+_T P_m)} (d_u + d_v)$$
$$+ \prod_{uv\in E_9(P_m+_T P_m)} (d_u + d_v) + \prod_{uv\in E_{10}(P_m+_T P_m)} (d_u + d_v)$$
$$= (5)^{|E_1(P_m+_T P_m)|} + (6)^{|E_2(P_m+_T P_m)|} + (7)^{|E_3(P_m+_T P_m)|} + (8)^{|E_4(P_m+_T P_m)|}$$
$$+(7)^{|E_5(P_m+_T P_m)|} + (8)^{|E_6(P_m+_T P_m)|} + (8)^{|E_7(P_m+_T P_m)|} + (10)^{|E_8(P_m+_T P_m)|}$$
$$+(10)^{|E_9(P_m+_T P_m)|} + (11)^{|E_{10}(P_m+_T P_m)|} = 6^4 + 7^{(8+4t)} + 8^8 + 9^{(2t)}$$
$$+8^{(2t^2+2t-6)} + (9)^4(t-1) + 10^{(2t^2)} + 10^2(t-1) + 11^2 t + 12^2 t(t-1)$$

(3). by definition

$$PM_1(P_m +_T P_m) = \prod_{uv\in E_1(P_m+_T P_m)} (d_u + \times) + \prod_{uv\in E_2(P_m+_T P_m)} (d_u \times d_v)$$
$$+ \prod_{uv\in E_3(P_m+_T P_m)} (d_u \times d_v) + \prod_{uv\in E_4(P_m+_T P_m)} (d_u \times d_v) + \prod_{uv\in E_5(P_m+_T P_m)} (d_u + \times)$$
$$+ \prod_{uv\in E_6(P_m+_T P_m)} (d_u \times d_v) + \prod_{uv\in E_7(P_m+_T P_m)} (d_u \times d_v) + \prod_{uv\in E_8(P_m+_T P_m)} (d_u \times d_v)$$
$$+ \prod_{uv\in E_9(P_m+_T P_m)} (d_u + \times) + \prod_{uv\in E_{10}(P_m+_T P_m)} (d_u \times d_v).$$

$$= (9)^{|E_1(P_m+_T P_m)|} + (12)^{|E_2(P_m+_T P_m)|} + (15)^{|E_3(P_m+_T P_m)|} + (18)^{|E_4(P_m+_T P_m)|}$$
$$+(16)^{|E_5(P_m+_T P_m)|} + (20)^{|E_6(P_m+_T P_m)|} + (24)^{|E_7(P_m+_T P_m)|} + (25)^{|E_8(P_m+_T P_m)|}$$
$$+(30)^{|E_9(P_m+_T P_m)|} + (36)^{|E_{10}(P_m+_T P_m)|}$$
$$= 9^4 + 12^{(8+4t)} + 15^8 + 18^{(2t)} + 16^{(2t^2+2t-6)}$$
$$+(20)^4(t-1) + 24^{(2t^2)} + 25^2(t-1) + 30^2 t + 36^2 t(t-1)$$

(4).

$$F(P_m +_T P_m) = \prod_{uv\in E_1(P_m+_T P_m)} (3^2+3^2) + \prod_{uv\in E_2(P_m+_T P_m)} (3^2+4^2)$$
$$+ \prod_{uv\in E_3(P_m+_T P_m)} (3^2+5^2) + \prod_{uv\in E_4(P_m+_T P_m)} (3^2+6^2) + \prod_{uv\in E_5(P_m+_T P_m)} (4^2+4^2)$$
$$+ \prod_{uv\in E_6(P_m+_T P_m)} (4^2+5^2) + \prod_{uv\in E_7(P_m+_T P_m)} (4^2+6^2) + \prod_{uv\in E_8(P_m+_T P_m)} (5^2+5^2)$$
$$+ \prod_{uv\in E_9(P_m+_T P_m)} (5^2+6^2) + \prod_{uv\in E_{10}(P_m+_T P_m)} (6^2+6^2)$$
$$= 18^4 + 25^{(8+4t)} + 34^8 + 45^{(2t)} + 32^{(2t^2+2t-6)}$$
$$+(41)^4(t-1) + 52^{(2t^2)} + 50^2(t-1) + 61^2 t + 72^2 t(t-1).$$

3D plot for hyper Zagreb, first Zagreb, second Zagreb and Forgotten indices are shown in Figure 8.

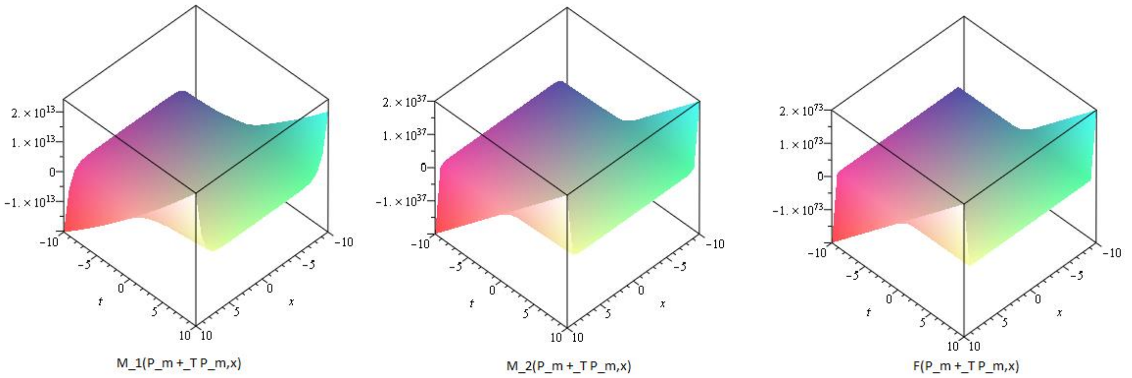

FIGURE 7 Graph of Algebraic polynomials for $P_m+_TP_m$

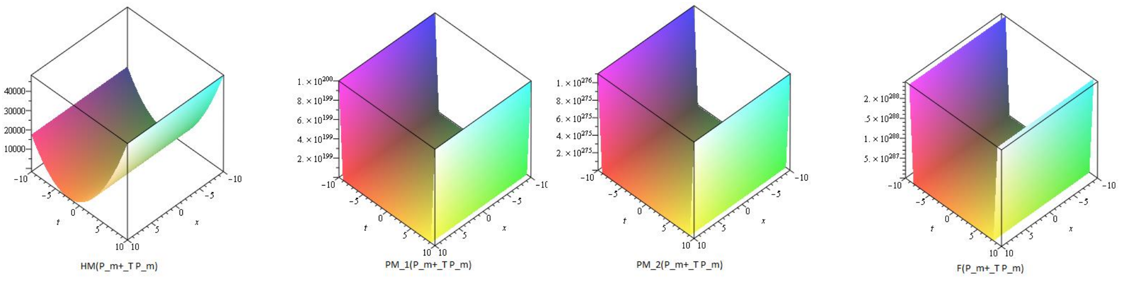

FIGURE 8 Graph of topological indices for ($P_m+_TP_m$)

Conclusion

In this article, we computed first Zagreb polynomial, second Zagreb polynomial and forgotten polynomial, and also closed forms of topological indices such as hyper Zagreb index, first Zagreb index, second Zagreb index and forgotten index for $P_m+_FP_m$ graphs.

Acknowledgments

This research is partially supported by COMSATS Attock and National University of Sciences and Technology, Islamabad, Pakistan and the Start-up Research Grant 2016 of United Arab Emirates University (UAEU), Al Ain, United Arab Emirates via Grant No. G00002233 and UPAR Grant of UAEU *via* Grant No. G00002590.

Orcid:
Abdul Qudair Baig:
https://orcid.org/0000-0002-8913-5221
Adnan Amin:
https://orcid.org/0000-0002-0852-8833
Mohammad Reza Farahani:
https://orcid.org/0000-0003-2185-5967
Muhammad Imran:
https://orcid.org/0000-0002-2827-0462
Murat Cancan:
https://orcid.org/0000-0002-8606-2274
Mehmet Serif Aldemir:
https://orcid.org/0000-0002-5298-6743

References

[1] G.H. Shirdel, H. Rezapour, A.M. Sayadi. Iranian, *J. Math. Chem*., **2013**, *4*, 213-220.
[2] M. Ghorbani, N. Azimi, *Iran. J. Math. Chem*., **2012**, *3*, 137-143.
[3] B. Furtula, I. Gutman, *J. Math. Chem*., **2015**, *53*, 11841190.
[4] I. Gutman, N. Trinajstić, *Chem. Phys. Lett*., **1972**, *17*, 535-538.
[5] A. Mukhtar, R. Murtaza, A.Q. Baig, M.R. Azhar, T. Zhao, *J. Inf. Opt. Sci*., **2020**, *41*, 1077–1091.
[6] S. Hayat, M. Imran, *Appl. Math. Comput*., **2014**, *240*, 213-228.
[7] A.Q. Baig, M. Imran, H. Ali, *Can. J. Chem*. **2015**, *93*, 730-739.
[8] A.Q. Baig, M. Imran, W. Khalid, M. Naeem, *Can. J. Chem*., **2017**, *95*, 674-686.
[9] X. Zhang, A. Razzaq, K. Ali, S.T.R. Rizvi, M.R. Farahani, *J. Inf. Opt. Sci*., **2020**, *41*, 865–877.
[10] M.S. Sardar, S.A. Xu, W. Sajjad, S. Zafar, I. N. Cangul, M.R. Farahani, *J. Inf. Opt. Sci*., **2020**, *41*, 879–890
[11] M.N. Husin, F. Asif, Z. Zahid, S. Zafar, *J. Inf. Opt. Sci*., **2020**, *41*, 891–903.
[12] Z. Ahmad, A.Q. Baig, M.R. Azhar, M. Imran, *J. Inf. Opt. Sci*., **2020**, *41*, 905–924.
[13] M. Cancan, S. Ediz, S. Fareed, M.R. Farahani, *J. Inf. Opt. Sci*., **2020**, *41*, 925–932.
[14] M. Alaeiyan, F. Afzal, M.R. Farahani, M.A. Rostami, *J. Inf. Opt. Sci*., **2020**, *41*, 933–939.
[15] M. N. Husin, A. Ariffin, M. Alaeiyan, *J. Inf. Opt. Sci*., **2020**, *41*, 941–948.
[16] M. Cancan, S. Ediz, M. Alaeiyan, M.R. Farahani, *J. Inf. Opt. Sci*., **2020**, *41*, 949–957.
[17] W. Sajjad, M. Sh. Sardar, M. Cancan, S. Ediz, A.Q. Baig, *J. Inf. Opt. Sci*., **2020**, *41*, 959–972.
[18] Amer, M. Naeem, H. Ur Rehman, M. Irfan, M.S. Saleem, H. Yang, *J. Inf. Opt. Sci*., **2020**, *41*, 973–990.
[19] M. Cancan, M. Naeem, A.Q. Baig, W. Gao, A. Aslam, *J. Inf. Opt. Sci*., **2020**, *41*, 991–1006.
[20] M. Cancan, M. Naeem, A. Aslam, Wei Gao, A.Q. Baig, *J. Inf. Opt. Sci*., **2020**, *41*, 1007–1024.
[21] M. Imran, M. Naeem, A.Q. Baig, *J. Inf. Opt. Sci*., **2020**, *41*, 1025–1041.
[22] F. Afzal, S. Hussain, D. Afzal, S. Razaq, *J. Inf. Opt. Sci*., **2020**, *41*, 1061–1076.
[23] U.K. Faraji, M. Alaeiyan, M. Golriz, A.R. Gilani, *J. Inf. Opt. Sci*., **2020**, *41*, 1077–1091.
[24] M. Cancan, S. Ediz, H.M. Ur-Rehman, D. Afzal, *J. Inf. Opt. Sci*., **2020**, *41*, 1117–1131.

How to cite this article: Abdul Qudair Baig*, Adnan Amin, Mohammad Reza Farahani, Muhammad Imran, Murat Cancan, Mehmet Serif Aldemir. Some topological descriptors and algebraic polynomials of $P_m{+_F}P_m$. *Eurasian Chemical Communications*, 2020, 2(10), 1072-1081. **Link:** http://www.echemcom.com/article_118049.html